Using the Discriminant GREEN

1. Calculate the value of the discriminant for each of the following functions.

   a) $f(x) = x^2 + 8x + 3$

   b) $g(x) = 2x^2 - 3x + 4$

   c) $h(x) = -x^2 + 7x - 3$

   d) $j(x) = x^2 - 8x + 16$

   e) $k(x) = 2x - 3x^2 - 4$

2. What do your answers to question 1 tell you about the roots of each of the functions?

   a) _____

   b) _____

   c) _____

   d) _____

   e) _____

3. Find the values of k for which $x^2 + 6x + k = 0$ has two real solutions.

4. Find the value of t for which $2x^2 - 3x + t = 0$ has exactly one solution.

5. Find the range of values of k for which $3x^2 - 4x + k = 0$ has no real solutions.

6. Given that the function $f(x) = sx^2 + 8x + s$ has equal roots, find the value of the positive constant s.

7. The function $g(x) = x^2 + 3px + (14p - 3)$, where p is an integer, has two equal roots.
   a) Find the value of p.

   b) For this value of p, solve the equation $x^2 + 3px + (14p - 3) = 0$.

8. $h(x) = 2x^2 + (k + 4)x + k$, where k is a real constant.
   a) Find the discriminant of $h(x)$ in terms of k.

   b) Hence or otherwise, prove that $h(x)$ has two distinct real solutions for all values of k.

Using the Discriminant AMBER

1. Calculate the value of the discriminant for each of the following functions.

$b^2 - 4ac$

a) $f(x) = x^2 + 8x + 3$

b) $g(x) = 2x^2 - 3x + 4$

c) $h(x) = -x^2 + 7x - 3$

d) $j(x) = x^2 - 8x + 16$

e) $k(x) = 2x - 3x^2 - 4$

2. What do your answers to question 1 tell you about the roots of each of the functions?

Hint: are the answers positive or negative?

a) _____

b) _____

c) _____

d) _____

e) _____

3. Find the values of k for which $x^2 + 6x + k = 0$ has two real solutions.

Hint: Two roots → $b^2 - 4ac > 0$

4. Find the value of t for which $2x^2 - 3x + t = 0$ has exactly one solution.

Hint: One roots → $b^2 - 4ac = 0$

5. Find the range of values of k for which $3x^2 - 4x + k = 0$ has no real solutions.

Hint: No roots → $b^2 - 4ac < 0$

6. Given that the function $f(x) = sx^2 + 8x + s$ has equal roots, find the value of the positive constant s.

Hint: One roots → $b^2 - 4ac = 0$

7. The function $g(x) = x^2 + 3px + (14p - 3)$, where p is an integer, has two equal roots.

a) Find the value of p.

Hint: Two roots → $b^2 - 4ac > 0$

b) For this value of p, solve the equation $x^2 + 3px + (14p - 3) = 0$.

8. $h(x) = 2x^2 + (k + 4)x + k$, where k is a real constant.

a) Find the discriminant of $h(x)$ in terms of k.

b) Hence or otherwise, prove that $h(x)$ has two distinct real solutions for all values of k.

Using the Discriminant RED

1. Calculate the value of the discriminant for each of the following functions.

b^2 - 4ac

a) $f(x) = x^2 + 8x + 3$ $\quad b^2 - 4ac = 8^2 - 4 \times 1 \times 3 =$

b) $g(x) = 2x^2 - 3x + 4$ $\quad b^2 - 4ac =$

c) $h(x) = -x^2 + 7x - 3$

d) $j(x) = x^2 - 8x + 16$

e) $k(x) = 2x - 3x^2 - 4$

2. What do your answers to question 1 tell you about the roots of each of the functions?

Hint: are the answers positive or negative?

a) Positive discriminant means f(x) has two real roots

b) _____

c) _____

d) _____

e) _____

3. Find the values of k for which $x^2 + 6x + k = 0$ has two real solutions.

$6^2 - 4 \times 1 \times k > 0$

$36 - 4k > 0$

Hint: Two roots → b^2 - 4ac > 0

Hint: Solve to find value of k

4. Find the value of t for which $2x^2 - 3x + t = 0$ has exactly one solution.

Hint: One roots → b^2 - 4ac = 0

5. Find the range of values of k for which $3x^2 - 4x + k = 0$ has no real solutions.

Hint: No roots → b^2 - 4ac < 0

6. Given that the function $f(x) = sx^2 + 8x + s$ has equal roots, find the value of the positive constant s.

Hint: One roots → $b^2 - 4ac = 0$

$8^2 - 4 \times s \times s = 0$

$64 - 4s^2 = 0$

7. The function $g(x) = x^2 + 3px + (14p - 3)$, where p is an **integer**, has two equal roots.

a) Find the value of p.

Hint: Two roots → $b^2 - 4ac > 0$

$(3p)^2 - 4(14p - 3) > 0$

b) For this value of p, solve the equation $x^2 + 3px + (14p - 3) = 0$.

8. $h(x) = 2x^2 + (k + 4)x + k$, where k is a real constant.

a) Find the discriminant of $h(x)$ in terms of k.

$a = 2, b = (k + 4), c = k$

b) Hence or otherwise, prove that $h(x)$ has two distinct real solutions for all values of k.

Using the Discriminant ANSWERS

1. Calculate the value of the discriminant for each of the following functions.

a) $f(x) = x^2 + 8x + 3$ $8^2 - 4 \times 1 \times 3 = 64 - 12 = 52$

b) $g(x) = 2x^2 - 3x + 4$ $(-3)^2 - 4 \times 2 \times 4 = 9 - 32 = -23$

c) $h(x) = -x^2 + 7x - 3$ $7^2 - 4 \times -1 \times -3 = 49 - 12 = 37$

d) $j(x) = x^2 - 8x + 16$ $(-8)^2 - 4 \times 1 \times 16 = 64 - 64 = 0$

e) $k(x) = 2x - 3x^2 - 4$ $2^2 - 4 \times -3 \times -4 = 4 - 48 = -44$

2. What do your answers to question 1 tell you about the roots of each of the functions?

a) Two real roots

b) No real roots

c) Two real roots

d) One repeated root

e) No real roots

3. Find the values of k for which $x^2 + 6x + k = 0$ has two real solutions.

$$b^2 - 4ac > 0$$
$$6^2 - 4 \times 1 \times k > 0$$
$$-4k > -36$$
$$k < 9$$

4. Find the value of t for which $2x^2 - 3x + t = 0$ has exactly one solution.

$$b^2 - 4ac = 0$$
$$(-3)^2 - 4 \times 2 \times t = 0$$
$$-8t = -9$$
$$t = \frac{9}{8}$$

5. Find the range of values of k for which $3x^2 - 4x + k = 0$ has no real solutions.

$$b^2 - 4ac < 0$$
$$(-4)^2 - 4 \times 3 \times k < 0$$
$$-12k < -16$$
$$k > \frac{4}{3}$$

6. Given that the function $f(x) = sx^2 + 8x + s$ has equal roots, find the value of the positive constant s.

$b^2 - 4ac = 0$

$8^2 - 4 \times s \times s = 0$

$-4s^2 = -64$

$s^2 = 16$

$s = \pm 4$

$s > 0 \therefore s = 4$

7. The function $g(x) = x^2 + 3px + (14p - 3)$, where p is an integer, has two equal roots.

a) Find the value of p.

$b^2 - 4ac = 0$

$(3p)^2 - 4 \times 1 \times (14p - 3) = 0$

$9p^2 - 56p + 12 = 0$

$(9p - 2)(p - 6) = 0$

$p = 6$ or $\frac{2}{9}$

$p \in \mathbb{Z} \therefore p = 6$

b) For this value of p, solve the equation $x^2 + 3px + (14p - 3) = 0$.

$x^2 + 18x + 81 = 0$

$(x + 9)^2 = 0$

$x = -9$

8. $h(x) = 2x^2 + (k + 4)x + k$, where k is a real constant.

a) Find the discriminant of $h(x)$ in terms of k.

$b^2 - 4ac = (k + 4)^2 - 4 \times 2 \times k$

$= k^2 + 8k + 16 - 8k$

$= k^2 + 16$

b) Hence or otherwise, prove that $h(x)$ has two distinct real solutions for all values of k.

Since $k^2 \geq 0$, $k^2 + 16 \geq 16 \therefore b^2 - 4ac \geq 16$

The discriminant will be positive for all values of k so $h(x)$ has two distinct real solutions.