

# Maths Problem Solving Starters

Levels 3 – 5

Name: Worked solutions

Class: \_\_\_\_\_

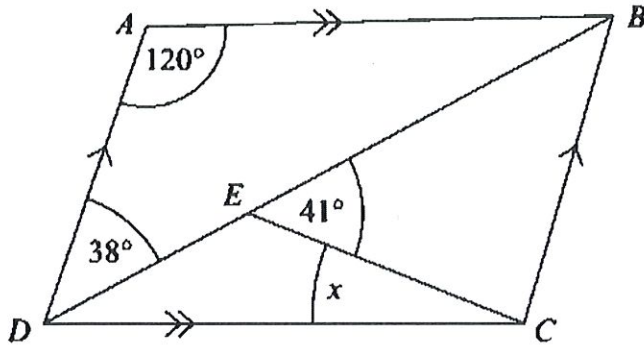
Teacher: \_\_\_\_\_

## Remember:

- You will need: pen, pencil, rubber and a ruler.
- Check your work carefully.
- Show all of your working out, with clear steps.

1.	<p>Susan did a puzzle in 5 minutes and 55 seconds. Jamal did the same puzzle in 11 minutes and 42 seconds. Susan says, 'I did the puzzle in less than half the time Jamal did the puzzle. Is Susan right? You must show all your working.</p> <p>Half of 11 mins 42 secs = 5 mins 51 secs.  <span style="color: red;">①</span></p> <p>Susan is wrong.  <span style="color: red;">①</span></p>	/ 3
2.	<p>Terry is going to make some concrete mix. He needs to mix cement, sand and gravel in the ratio 2 : 3 : 5 by weight. He estimates he needs to make 220 kg of concrete mix.</p> <p>Terry has:</p> <ul style="list-style-type: none"> <li>• 50 kg of cement</li> <li>• 90 kg of sand</li> <li>• 100 kg of gravel</li> </ul> <p>Does Terry have enough cement, sand and gravel to make the concrete mix?</p> <p><math>220 \text{ kg} \div 10 = 22 \text{ kg}</math> (1 part). <span style="color: red;">①</span></p> <p>Cement: <math>2 \times 22 = 44 \text{ kg}</math> needed ✓</p> <p>Sand: <math>3 \times 22 = 66 \text{ kg}</math> needed ✓ <span style="color: red;">②</span></p> <p>Gravel: <math>5 \times 22 = 110 \text{ kg}</math> needed x.</p> <p>Terry has enough cement and sand, but not enough gravel. <span style="color: red;">①</span></p>	/ 4

3.

Diagram NOT  
accurately drawn

ABCD is a parallelogram. Angle  $ADB = 38^\circ$ . Angle  $BEC = 41^\circ$ . Angle  $DAB = 120^\circ$ . Calculate the size of angle  $x$ . You must give reasons for your answer.

$ADC = 60^\circ$  because co-interior angles sum to  $180^\circ$ . ①

$$BDC = 60 - 38 = 22^\circ$$

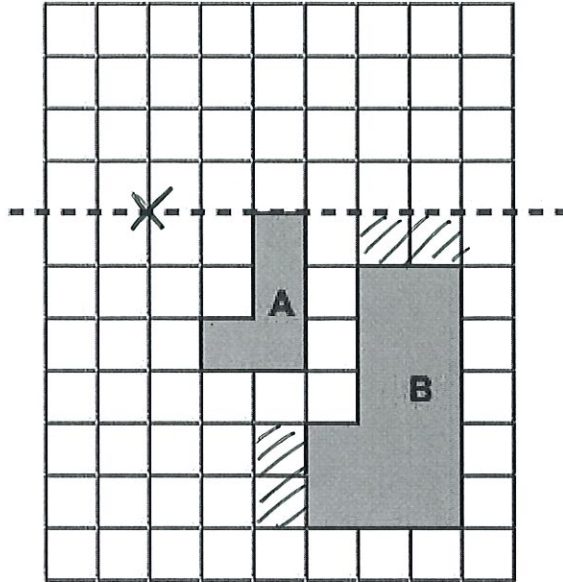
$DEC = 139^\circ$  because angles on a straight line sum to  $180^\circ$ . ①

$x = 180 - (22 + 139) = 19^\circ$  because angles in a triangle sum to  $180^\circ$ . ①

$$x = \underline{19}^\circ$$

4.

On this grid are two shapes, A and B. Shape B is an enlargement of shape A, but some parts of B are missing. The centre of enlargement is on the dotted line.



(a) Shade in squares to complete shape B. ②

(b) Find the centre of enlargement and mark it on the diagram with an 'X'. ①

(c) What is the scale factor of the enlargement? 2 ①

/ 4

5.

Salima sees an advery for a summer holiday:

### Summer Sun

#### Fantastic deals with Sunbreak Holidays

Dates	7 nights	14 nights
1 <sup>st</sup> April – 30 <sup>th</sup> April	<span style="border: 1px solid black; border-radius: 50%; padding: 2px;">£315</span>	£575
<del>1<sup>st</sup> May – 6<sup>th</sup> July</del>	<del>£220</del>	<del>£400</del>

**Notes:**

- Prices are for one adult (16 years and over)
- Children (less than 16 years) 75% of the price
- 10% discount if booked online ([www.sunbreaks.co.uk](http://www.sunbreaks.co.uk))

Salima books a 7 night holiday in April for 2 adults. The travel agent adds a percentage surcharge to the cost of the holiday for booking fees. Salima's final bill is £642.60. What was the percentage surcharge?



$$£315 \times 2 = £630 \quad \text{①}$$

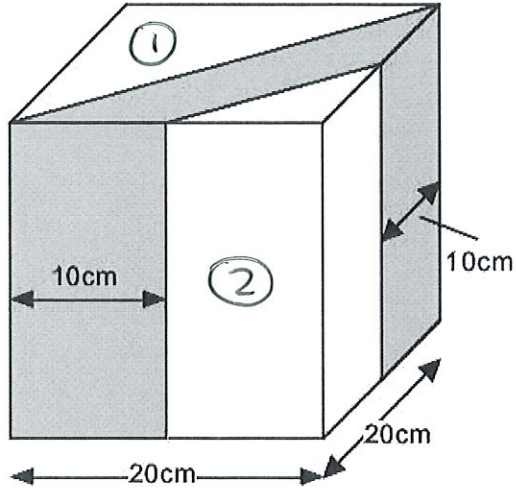
$$£642.60 - £630 = £12.60 \quad \text{①}$$

$$\frac{12.6}{630} \times 100 = 2\% \quad \text{①}$$

①  
2 %

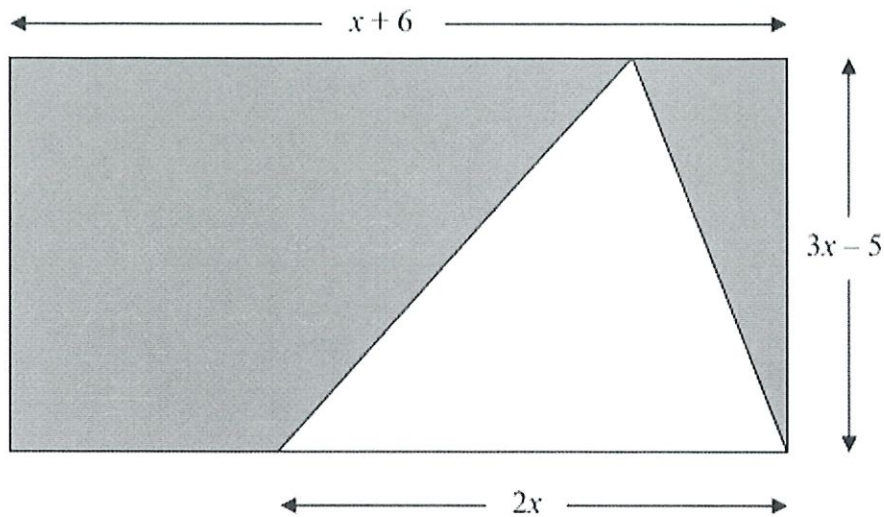
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6.	<p>A cube is cut into three parts by two vertical slices. Find the volume of the shaded part.</p>  <p>Volume of cube:  <math>= 20^3 = 8000 \text{ cm}^3</math> ①</p> <p>Volume of ①:  <math>= \frac{20^3}{2} = 4000 \text{ cm}^3</math> ①</p> <p>Volume of ②:  <math>= \frac{10 \times 10 \times 20}{2} = 1000 \text{ cm}^3</math> ①</p> <p><math>8000 - (4000 + 1000)</math> ①     <u>3000</u> ① <math>\text{ cm}^3</math></p>	15
7.	<p>Noah has some marbles. He gives Keira 20% of his marbles. He gives Ali 50% of his marbles. Noah has 18 marbles left. How many marbles did he have to start with?</p> <p><math>100\% - (50\% + 20\%) = 30\%</math> ① ①</p> <p>30% of <math>x = 18</math> ①</p> <p>100% of <math>x = 60</math> ①</p> <p style="text-align: right;"><u>60</u></p>	14
8.	<p>Janice has three coins in her pocket, and they are all different from each other. Jeremy has three coins in his pocket and they are all the same as each other. Jeremy has twice as much money as Janice. What are the coins they each have?</p> <p><math>(5p + 10p + 20p) \times 2 = 70p</math> (not <math>\div 3</math>)  <math>(5p + 10p + 50p) \times 2 = 130p</math> (not <math>\div 3</math>)  <math>(5p + 20p + 50p) \times 2 = 150p</math>. ✓ ① for attempt</p> <p>Janice     <u>5p</u>     <u>20p</u>     <u>50p</u>  Jeremy     <u>50p</u>     <u>50p</u>     <u>50p</u> ①</p>	12

9.

The diagram shows a triangle inside a rectangle.



All measurements are given in centimetres. Show that the total area, in  $\text{cm}^2$ , of the shaded regions is  $18x - 30$ .

$$\text{Area of rectangle: } (x+6)(3x-5) \quad (1)$$

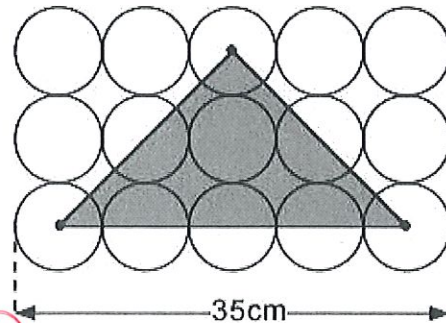
$$\text{Area of triangle: } \frac{2x(3x-5)}{2} \quad (1)$$

$$\begin{aligned} \text{Shaded area: } & 3x^2 + 13x - 30 - (3x^2 - 5x) \quad (1) \\ & = 18x - 30 \quad (1) \end{aligned}$$

15

10.

The diagram shows 15 identical circles, arranged as a rectangle, and a shaded triangle. The vertices of the triangle are at the centres of circles. Calculate the area of the shaded triangle.



$$\text{Diameter: } 35 \div 5 = 7\text{cm}$$

$$\text{Radius: } 7 \div 2 = 3.5\text{cm.} \quad (1)$$

$$\text{Base: } 8 \times 3.5 = 28\text{cm} \quad (1)$$

$$\text{Height: } 4 \times 3.5 = 14\text{cm} \quad (1)$$

$$\text{Area: } \frac{28 \times 14}{2} = 196\text{cm}^2 \quad (1)$$

$$\underline{196} \text{ cm}^2$$

15

11.

Three dice are each numbered 1 to 6. Two of them are red and one is blue. All three dice are rolled. What is the probability that the total on the two red dice will be equal to the score on the blue dice?

Red 1

	1	2	3	4	5	6
1	2	3	4	5	6	///
2	3	4	5	6	///	///
3	4	5	6	///	///	///
4	5	6	///	///	///	///
5	6	///	///	///	///	///
6	///	///	///	///	///	///

Red 2

②

$$P(\text{red 1}) \text{ and } P(\text{blue 1}) = 0 \times \frac{1}{6} = 0$$

$$P(\text{red 2}) \text{ and } P(\text{blue 2}) = \frac{1}{36} \times \frac{1}{6} = \frac{1}{216}$$

$$P(\text{red 3}) \text{ and } P(\text{blue 3}) = \frac{2}{36} \times \frac{1}{6} = \frac{2}{216}$$

$$P(\text{red 4}) \text{ and } P(\text{blue 4}) = \frac{3}{36} \times \frac{1}{6} = \frac{3}{216}$$

$$P(\text{red 5}) \text{ and } P(\text{blue 5}) = \frac{4}{36} \times \frac{1}{6} = \frac{4}{216}$$

$$P(\text{red 6}) \text{ and } P(\text{blue 6}) = \frac{5}{36} \times \frac{1}{6} = \frac{5}{216}$$

③

$$\frac{1}{216} + \frac{2}{216} + \frac{3}{216} + \frac{4}{216} + \frac{5}{216} = \frac{15}{216}$$

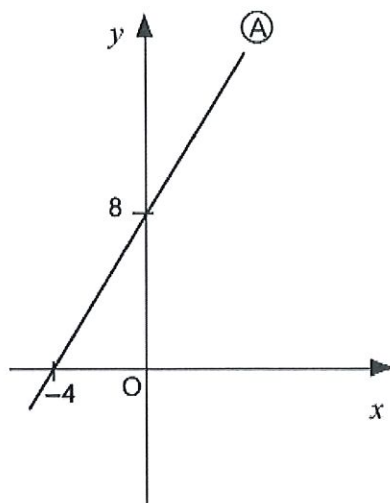
①

①

$$\frac{15}{216}$$



12. Here is a graph. What is the equation of the line?

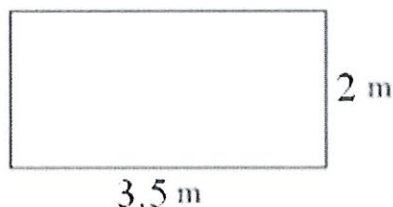


not drawn to scale

$$y = 2x + 8$$

12

13. The diagram shows a patio in the shape of a rectangle. The patio is 3.5 m long and 2m wide. Matt is going to cover the patio with paving slabs. Each paving slab is a square of side 50 cm. Matt buys 30 of the paving slabs. Does Matt buy enough paving slabs to cover the patio? You must show all your working.



$$2\text{m} \div 50\text{cm} = 4$$

$$3.5\text{m} \div 50\text{cm} = 7$$

$$4 \times 7 = 28 \text{ slabs needed.}$$

Yes Matt has enough.

14

14. Arnie saw a camera priced at £250 in London. He saw the same camera priced at \$297.50 in New York. This is a 30% saving on the London price. How many dollars are there to the pound?



$$£250 \div 0.7 = £357.14$$

$$£357.14 \div \$297.50 = 1.2004\dots$$

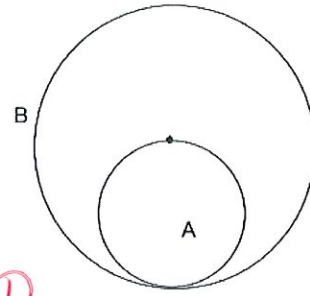
$$\$ 1.2$$

13



15.

The circumference of circle A touches the edge of circle B and passes through its centre. The area of circle A is  $100\text{cm}^2$ . What is the area of circle B?



$$\text{Linear scale factor} = 2 \quad (1)$$

$$\text{Area scale factor} = 2^2 = 4 \quad (1)$$

$$100 \times 4 = 400\text{cm}^2 \quad (1)$$

$$\underline{400} \text{ cm}^2$$

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16.

Five times a number gives the same answer as adding 24 to the number. What is the number?

$$5x = x + 24 \quad (1)$$

$$4x = 24 \quad (1)$$

$$x = 6 \quad (1)$$

$$\underline{6}$$

/ 3

17.

Peter goes for a walk. He walks  $17\frac{1}{2}$  miles in 5 hours.

(a) Work out Peter's average speed. Give your answer in miles per hour.

$$S = \frac{D}{T} = \frac{17.5}{5} = 3.5$$

3.5 mph

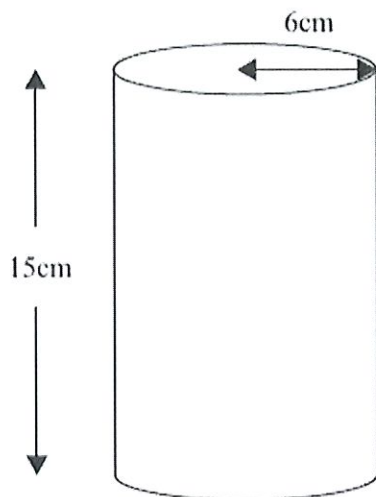
(b) Given 5 miles = 8 km, Sunita says that Peter walked more than 25 km. Is Sunita right?

$$17.5 \div 5 \times 8 = 28 \text{ km.}$$

Sunita is correct.

18.

Jenny fills some empty flowerpots completely with compost. Each flowerpot is in the shape of a cylinder of height 15 cm and radius 6 cm. She has a 15 litre



bag of compost. She fills up each flowerpot completely. How many flowerpots can she fill? You must show your working.

$$\begin{aligned} \text{Volume} &: 6^2 \times \pi \times 15 \\ &= 1696.46 \text{ cm}^3 \quad (1) \\ &= 1.70 \text{ L} \quad (1) \end{aligned}$$

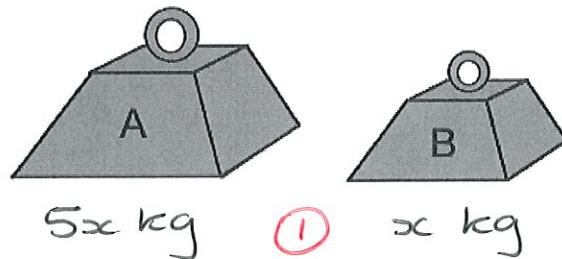
$$15 \div 1.70 = 8.841... \quad (1)$$

8 (1)

/ 4

19.

A and B are two weights. A is five times as heavy as B. The difference between the weights is 6kg. Find the weight of A.



$$5x - x = 6 \quad (1)$$

$$4x = 6$$

$$x = 1.5 \quad (1)$$

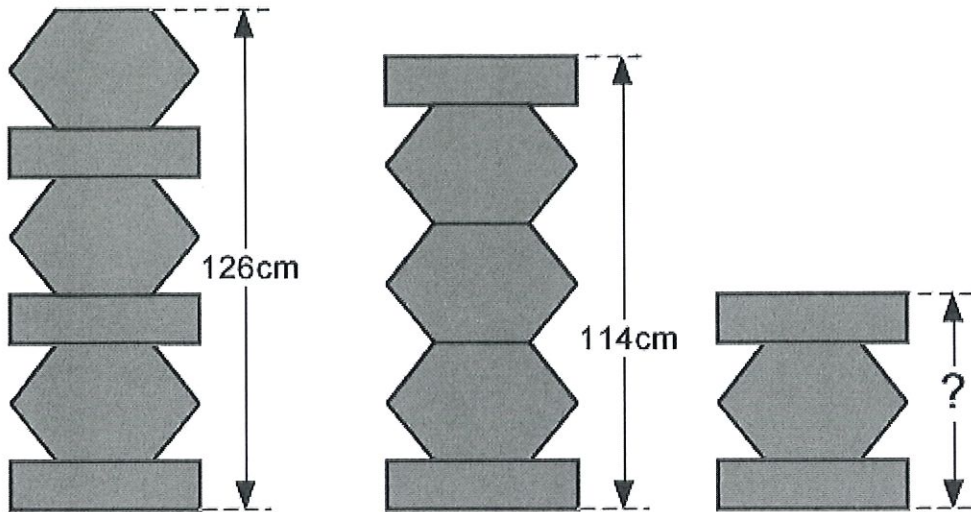
$$5x = 7.5 \quad (1)$$

7.5 kg

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20.

These towers are made of identical hexagons and identical rectangles.



Calculate the height of the smallest tower.

$$\begin{array}{r} 3r + 3h = 126 \\ - 2r + 3h = 114 \\ \hline r = 12 \text{ cm} \end{array}$$

$$\begin{array}{r} 3r + 3h = 126 \\ 36 + 3h = 126 \\ 3h = 90 \\ h = 30 \text{ cm} \end{array}$$

$$2r + h = 24 + 30 = 54 \text{ cm}$$

54 cm



21. Henry wants to invest £4000 for 2 years. He can choose between two different banks.

**Bank A**  
Earns 3% per annum compound interest

**Bank B**  
Earns 3.2% per annum simple interest  
Each year's interest is paid by cheque

Henry wants to earn as much interest on his investment as possible. Which bank should Henry choose? You must show your working.

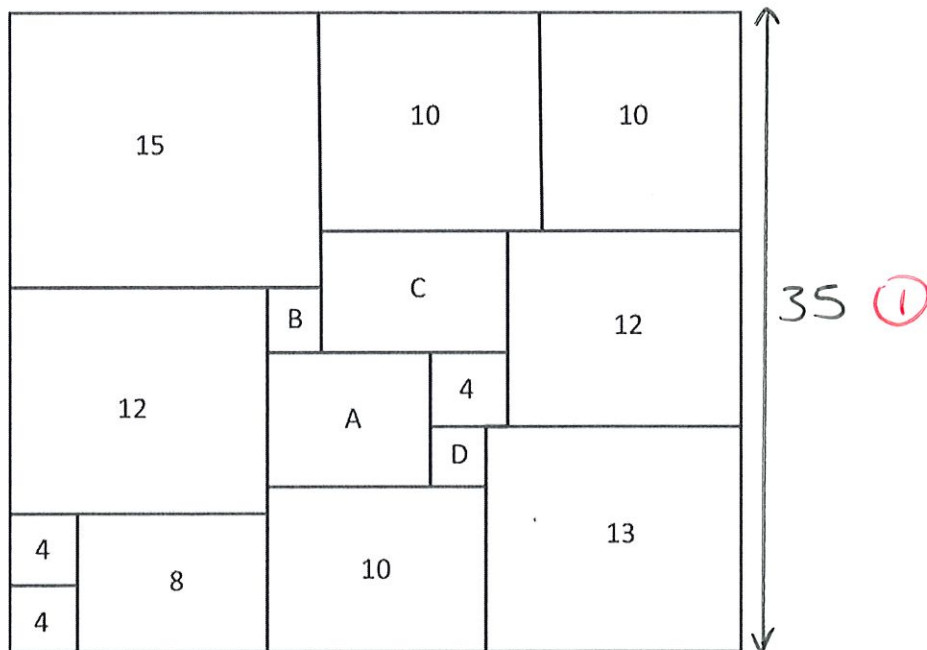
Bank A:  $4000 \times 1.03^2 = 4243.60$  (1)

Bank B:  $4000 \times 1.064 = 4256$  (1)

Henry should choose Bank B. (1)

15

22. Each shape contained within the largest square is also a square. The number in each square gives the length of its sides. What are the values of A, B, C and D?



A = 7 (1)    B = 3 (1)    C = 8 (1)    D = 3 (1)

15

23.

Mrs Binder owns a second hand book shop. She buys in 30 'good as new books' for £3 each. She chooses a selling price that will give her a 40% profit. She sells  $\frac{3}{5}$  of the books at this price. She then reduced the selling price by 20% and sold the remaining books at this price. Calculate Mrs Binder's overall percentage profit on this transaction.



$$\text{Spends: } 30 \times £3 = £90 \quad (1)$$

$$\text{New price: } 1.4 \times £3 = £4.20 \quad (1)$$

$$\frac{3}{5} \text{ of } 30 = 18 \text{ books} \quad (1)$$

$$18 \times £4.20 = \underline{£75.60} \quad (1)$$

$$0.8 \times £4.20 = £3.36$$

$$12 \times £3.36 = \underline{£40.32} \quad (1)$$

$$\begin{aligned} \text{Total takings: } & £75.60 + £40.32 \\ & = £115.92 \end{aligned}$$

$$£115.92 - £90 = £25.92 \quad (1)$$

$$\frac{25.92}{90} \times 100 = 28.8\% \text{ profit} \quad (1)$$

$$\underline{28.8} \quad \%$$

24.

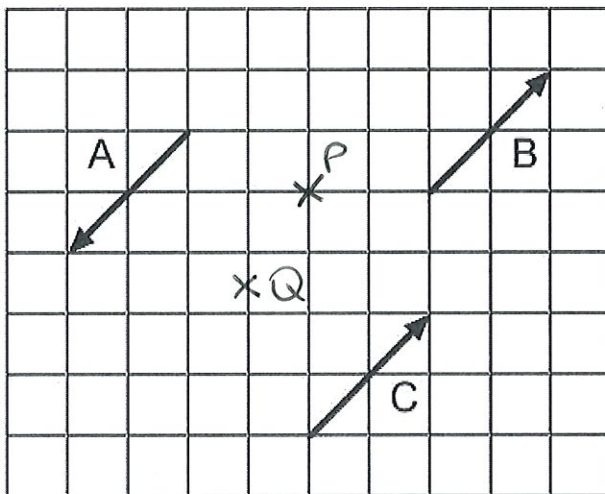
Jim's pay is £240 each week. Jim asks his boss for an increase of £40 a week. Jim's boss offers him a 15% increase. Is the offer from Jim's boss more than Jim asked for? You must show your working.

$$\begin{array}{r} 15\% \text{ of } \pounds 240 \\ \hline 10\% = \pounds 24 \\ 5\% = \pounds 12 \\ \hline \pounds 36 \end{array}$$

No, it is less than Jim asked for.

/ 3

25.

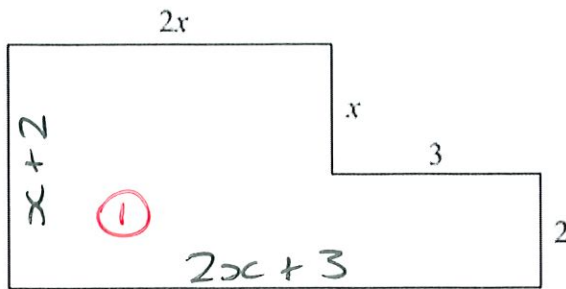


(a) The arrow in position A is rotated into position B. Mark the point P that is the centre of this rotation. (1)

(b) The arrow in position A is rotated into position C. Mark the point Q that is the centre of this rotation. (1)

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26.

Diagram NOT  
accurately drawn

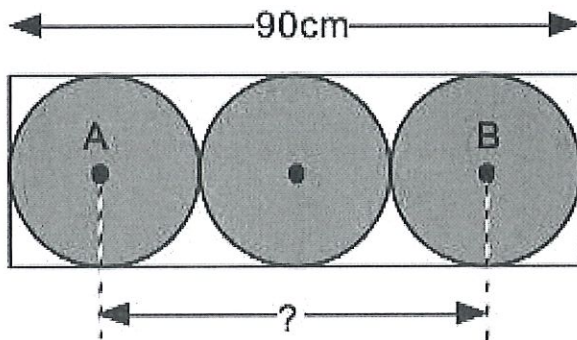
In the diagram, all measurements are given in centimetres. All angles are right angles. Show that the perimeter of the shape can be written as  $2(3x + 5)$ .

$$\begin{aligned}
 \text{Perimeter} &= 2x + x + 3 + 2 + 2x + 3 + x + 2 \\
 &= 6x + 10 \\
 &= 2(3x + 5)
 \end{aligned}$$

/ 4

27.

Three identical circles fit inside a rectangle. The length of the rectangle is 90cm.



Find the distance between the two centres, A and B.

$$\text{Diameter} : 90 \div 3 = 30 \text{ cm}$$

$$\text{Radius} : 30 \div 2 = 15 \text{ cm}$$

$$4 \times 15 = 60 \text{ cm}$$

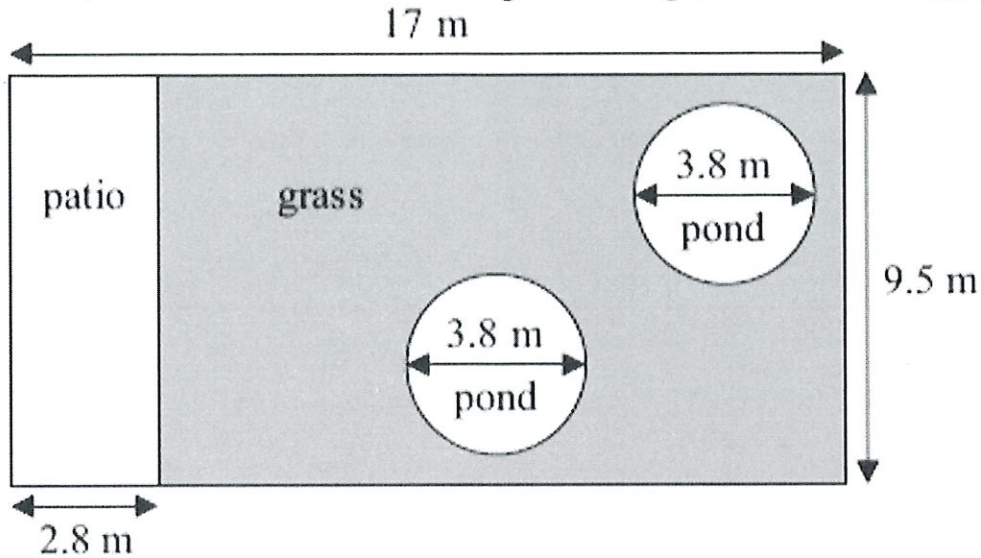
$$\underline{\quad 60 \quad} \text{ cm}$$

/ 3



28.

Mr Wells' garden is in the shape of a rectangle. In the garden there is a patio in the shape of a rectangle and two ponds in the shape of circles with diameter 3.8 m. The rest of the garden is grass.



Mr Wells' is going to spread fertiliser over all the grass. One bag of fertiliser will cover  $12 \text{ m}^2$  of grass. How many bags of fertiliser does Mr Wells need? You must show your working.

$$\begin{aligned} \text{Area: } & (14.2 \times 9.5) - (2 \times 1.9^2 \times \pi) \quad (1) \\ & = 134.9 - 22.68\dots \\ & = 112.21\dots \text{ m}^2 \quad (1) \end{aligned}$$

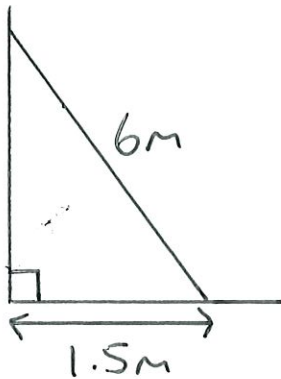
$$112.2 \div 12 = 9.35 \quad \therefore 10 \text{ bags needed.} \quad (1) \quad (1) \quad (1)$$

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 10

29.

A ladder is 6 m long. The ladder is placed on horizontal ground, resting against a vertical wall. The instructions for using the ladder say that the bottom of the ladder must not be closer than 1.5 m from the bottom of the wall. How far up the wall can the ladder reach? Give your answer correct to 1 decimal place.



$$\sqrt{6^2 - 1.5^2} \quad \textcircled{1}$$

$$= 5.809\dots \quad \textcircled{1}$$

$$\underline{\quad 5.8 \quad} \text{ m} \quad \textcircled{1}$$

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30.

The table below shows the change in the value of Seesaw plc shares over the last three years.

Year	2004	2005	2006
Change in value	+25%	-40%	+40%

*Note: the percentage change each year is based upon the value at the start of that year and the value at the end of that year*

Calculate the percentage change in Seesaw plc shares from the start of 2004 to the end of 2006.

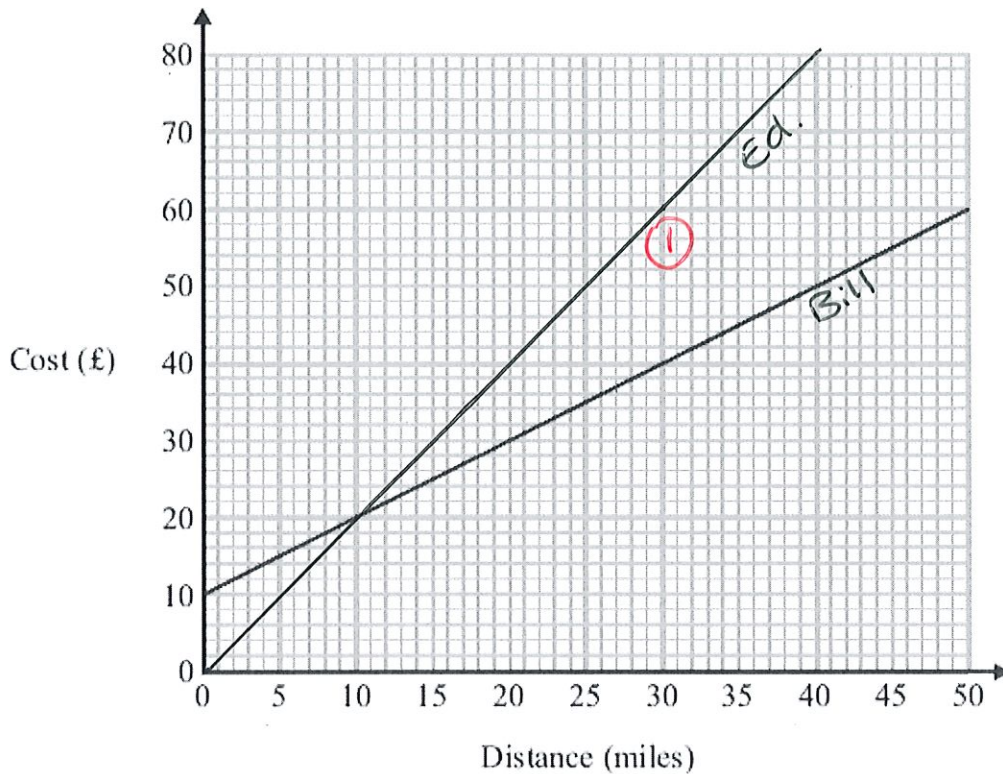
$$1.25 \times 0.6 \times 1.4 = 1.05 \quad \textcircled{1} \quad \textcircled{1}$$

$$\underline{\quad 5 \quad} \% \quad \underline{\quad \text{increase} \quad} \quad \textcircled{1}$$

/ 4

31.

Bill uses his van to deliver parcels. For each parcel Bill delivers there is a fixed charge plus £1.00 for each mile. You can use the graph to find the total cost of having a parcel delivered by Bill.



(a) How much is the fixed charge?

£ 10 ①

Ed uses a van to deliver parcels. For each parcel Ed delivers it costs £2 for each mile. There is no fixed charge.

(b) Use the graph to compare the cost of having a parcel delivered by Bill with the cost of having a parcel delivered by Ed.

Ed is cheaper for less than 10 miles. ①

Bill is cheaper for more than 10 miles. ①



32.

$\sqrt{20\,000} = 141.4$  (correct to 1 decimal place)  
 What is the smallest whole number that has a square root equal to 141.4 (correct to 1 decimal place)?



Lower bound = 141.35 (1)

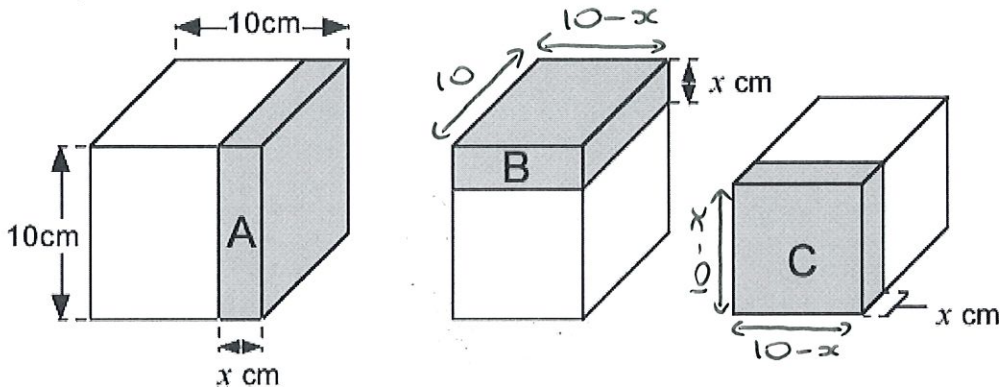
$141.35^2 = 19979.8225$  (1)

19980 (1)

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33.

A cube has edges of 10cm each. Three slices, each of thickness  $x$  cm, are cut off the cube.



Slice A is cut off the side, slice B is cut off the top and slice C is cut off the front. What is the volume of each slice in terms of  $x$ ?

A:  $10 \times 10 \times x = 100x$  (1)

B:  $10 \times x \times (10 - x) = 10x(10 - x)$   
 $= 100x - 10x^2$  (1)

C:  $x(10 - x)(10 - x) = x(100 - 20x + x^2)$  (1)  
 $= 100x - 20x^2 + x^3$  (1)

Slice A = 100x cm<sup>3</sup>

Slice B = 100x - 10x<sup>2</sup> cm<sup>3</sup>

Slice C = 100x - 20x<sup>2</sup> + x<sup>3</sup> cm<sup>3</sup>

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34.

The Hawshaw Summer Fete is running a competition. You buy a scratch card with 9 squares covered up. Under the 9 squares on each card, randomly placed are 4 stars, 3 hearts and 2 LOSE.



Hawshaw Summer Fete		
★	LOSE	♥
★	♥	★
LOSE	★	♥

Each scratch card costs £1. You scratch off two squares. You win £1.50 if 2 stars are revealed. You win £2 if 2 hearts are revealed.

Michelle buys a scratch card. Work out the probability that this will be a winning scratch card.

$$P(2 \text{ stars}) = \frac{4}{9} \times \frac{3}{8} = \frac{12}{72} \quad (1)$$

$$P(2 \text{ hearts}) = \frac{3}{9} \times \frac{2}{8} = \frac{6}{72} \quad (1)$$

$$\frac{12}{72} + \frac{6}{72} = \frac{18}{72} \quad (1)$$

$$\frac{18}{72} = \frac{1}{4} \quad (1)$$

There are 1440 scratch cards sold at the Fete. All of the proceeds go to charity. Estimate the amount of money raised for charity.

$$1440 \times £1 = £1440 \text{ received}$$

$$\frac{12}{72} \times 1440 \times £1.50 = £360 \text{ winnings} \quad (1)$$

$$\frac{6}{72} \times 1440 \times £2 = £240 \text{ winnings} \quad (1)$$

$$£1440 - (£360 + £240) = £840 \quad (1)$$

$$£ \underline{840}$$

35.

A coffee machine dispenses 130 ml of black coffee into cups with a capacity of 175 ml. These values are accurate to 3 significant figures. Milk is supplied in small cartons which contain 21 ml, accurate to the nearest ml. Beryl likes milky coffee and always puts 2 cartons of milk in her coffee. Will Beryl's cup ever overflow? You must show your working.

$$129.5 \leq \text{coffee} < 130.5 \quad (1)$$

$$174.5 \leq \text{cups} < 175.5 \quad (1)$$

$$20.5 \leq \text{milk} < 21.5 \quad (1)$$

$$\begin{aligned} \text{Upper bound} &: 130.5 + 2 \times 21.5 \quad (1) \\ &= 173.5 \text{ ml} \quad (1) \end{aligned}$$

Beryl's cup will not overflow. (1)