Using Properties and Equations of Circles GREEN

1. The points $U(-2,8)$, $V(7,7)$ and $W(-3,-1)$ lie on a circle.
   a) Show that the triangle UVW has a right angle.

   b) Write down an equation for the circle.

2. The points $R(-2,1)$, $S(4,3)$ and $T(10,-5)$ lie on the circumference of a circle. Find an equation of the circle.

3. Consider the points $A(3, 15)$, $B(-13, 3)$, $C(-7, -5)$ and $D(8, 0)$.

a) Show that ABC is a right-angled triangle.

b) Find the equation of the circumcircle.

c) Hence show that A, B, C and D all lie on the circumference of this circle.

4. A circle has equation $x^2 + 2x + y^2 - 24y - 24 = 0$

a) Find the centre and radius of the circle.

The points $A(-13, 17)$ and $B(11, 7)$ both lie on the circumference of the circle.

b) Show that AB is a diameter of the circle.

The point C lies on the negative x-axis and the angle $ACB = 90º$.

c) Find the coordinates of C.

Using Properties and Equations of Circles AMBER

1. The points $U(-2,8)$, $V(7,7)$ and $W(-3,-1)$ lie on a circle.

   a) Show that the triangle UVW has a right angle.

Start by calculating the gradients of all 3 lines. Are any perpendicular?

   b) Write down an equation for the circle.

Calculate midpoint of diameter to find centre of circle.

2. The points $R(-2,1)$, $S(4,3)$ and $T(10,-5)$ lie on the circumference of a circle. Find an equation of the circle.

For equation of a circle, we need length of radius and centre point. We can use the intersection of perpendicular bisectors to find the centre. The length of the radius will be the distance from the centre to any vertex of the triangle.

3. Consider the points $A(3, 15)$, $B(-13, 3)$, $C(-7, -5)$ and $D(8, 0)$.

a) Show that ABC is a right-angled triangle.

Start by calculating the gradients of all 3 lines. Are any perpendicular?

b) Find the equation of the circumcircle.

Calculate midpoint of diameter to find centre of circle.

c) Hence show that A, B, C and D all lie on the circumference of this circle.

Substitute D into the equation of the circle.

4. A circle has equation $x^2 + 2x + y^2 - 24y - 24 = 0$

a) Find the centre and radius of the circle.

Complete the square for x and y.

The points $A(-13, 17)$ and $B(11, 7)$ both lie on the circumference of the circle.

b) Show that AB is a diameter of the circle.

Calculate midpoint of diameter to find centre of circle.

The point C lies on the negative x-axis and the angle $ACB = 90º$.

c) Find the coordinates of C.

Substitute C into the equation of the circle.

U**sing Properties and Equations of Circles RED**

1. The points $U(-2,8)$, $V(7,7)$ and $W(-3,-1)$ lie on a circle.

a) Show that the triangle UVW has a right angle.

Start by calculating the gradients of all 3 lines. Are any perpendicular?

Gradient UV = _____ =

Gradient VW = _____ =

Gradient UW = _____ =

b) Write down an equation for the circle.

Calculate midpoint of diameter to find centre of circle.

Midpoint ____ =

(x)² + (y)² =

2. The points $R(-2,1)$, $S(4,3)$ and $T(10,-5)$ lie on the circumference of a circle. Find an equation of the circle.

For equation of a circle, we need length of radius and centre point. We can use the intersection of perpendicular bisectors to find the centre. The length of the radius will be the distance from the centre to any vertex of the triangle.

Midpoint RS =

Gradient ST = _____ =

Midpoint ST =

3. Consider the points $A(3, 15)$, $B(-13, 3)$, $C(-7, -5)$ and $D(8, 0)$.

a) Show that ABC is a right-angled triangle.

Start by calculating the gradients of all 3 lines. Are any perpendicular?

b) Find the equation of the circumcircle.

Calculate midpoint of diameter to find centre of circle.

c) Hence show that A, B, C and D all lie on the circumference of this circle.

Substitute D into the equation of the circle.

4. A circle has equation $x^2 + 2x + y^2 - 24y - 24 = 0$

a) Find the centre and radius of the circle.

Complete the square for x and y.

The points $A(-13, 17)$ and $B(11, 7)$ both lie on the circumference of the circle.

b) Show that AB is a diameter of the circle.

Calculate midpoint of diameter to find centre of circle.

The point C lies on the negative x-axis and the angle $ACB = 90º$.

c) Find the coordinates of C.

Substitute C into the equation of the circle.

Using Properties and Equations of Circles ANSWERS

1. The points $U(-2,8)$, $V(7,7)$ and $W(-3,-1)$ lie on a circle.

a) Show that the triangle UVW has a right angle.

Gradient of $UV = \frac{7-8}{7--2} = -\frac{1}{9}$

Gradient of $UW = \frac{-1-8}{-3--2} = 9$

Since UV and UW are perpendicular $\left(-\frac{1}{9} \times 9 = -1\right)$,

$\angle VUW = 90°$.

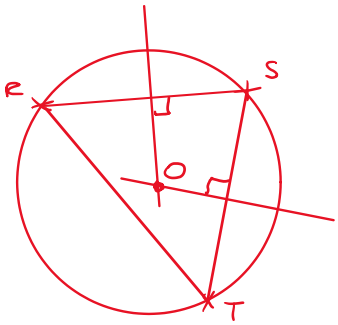
b) Write down an equation for the circle.

VW is diameter

Midpoint: $(2, 3)$ and radius $= \sqrt{41}$

$(x-2)^2 + (y-3)^2 = 41$

2. The points $R(-2,1)$, $S(4,3)$ and $T(10,-5)$ lie on the circumference of a circle. Find an equation of the circle.



Gradient of $RS = \frac{3-1}{4--2} = \frac{1}{3}$

Midpoint of $RS = (1, 2)$

$y - 2 = -3(x-1) \Rightarrow y = -3x + 5$

Gradient of $ST = \frac{-5-3}{10-4} = -\frac{4}{3}$

Midpoint of $ST = (7, -1)$

$y + 1 = \frac{3}{4}(x-7) \Rightarrow y = \frac{3}{4}x - \frac{25}{4}$

Equate: $\frac{3}{4}x - \frac{25}{4} = -3x + 5 \Rightarrow x = 3, y = -4$

Radius: $\sqrt{50}$

Equation of circle: $(x-3)^2 + (y+4)^2 = 50$

3. Consider the points $A(3,15)$, $B(-13,3)$, $C(-7,-5)$ and $D(8,0)$.

a) Show that ABC is a right-angled triangle.

Gradient of AB $= \frac{3-15}{-13-3} = \frac{3}{4}$

Gradient of BC $= \frac{-5-3}{-7--13} = -\frac{4}{3}$

Since AB and BC are perpendicular, $\angle ABC = 90°$

b) Find the equation of the circumcircle.

Midpoint of AC $= (-2, 5)$

Radius $= \sqrt{125}$

$(x+2)^2 + (y-5)^2 = 125$

c) Hence show that A, B, C and D all lie on the circumference of this circle.

$D(8,0) \Rightarrow (8+2)^2 + (0-5)^2 = 10^2 + 5^2 = 125$

D satisfies the equation of the circle so also lies on the circumference.

4. A circle has equation $x^2 + 2x + y^2 - 24y - 24 = 0$

a) Find the centre and radius of the circle.

$(x+1)^2 - 1 + (y-12)^2 - 144 - 24 = 0$

$(x+1)^2 + (y-12)^2 = 169$

Centre $(-1, 12)$ and radius 13

The points $A(-13,17)$ and $B(11,7)$ both lie on the circumference of the circle.

b) Show that AB is a diameter of the circle.

Midpoint AB $= (-1, 12)$ which is centre of circle.

$|AB| = \sqrt{24^2 + 10^2} = \sqrt{676} = 26$ which is double the radius.

The point C lies on the negative x-axis and the angle $ACB = 90º$.

c) Find the coordinates of C.

$C(x, 0) \Rightarrow (x+1)^2 + (-12)^2 = 169$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = -6$ or 4

$x < 0 \therefore x = -6. \Rightarrow C(-6, 0)$