Trigonometric Identities ANSWERS

1. Simplify each of the following expressions:

a. $1 - \cos^2 \tfrac{1}{2}\theta$

$\equiv \sin^2 \tfrac{1}{2}\theta$

b. $5\sin^2 3\theta + 5\cos^2 3\theta$

$\equiv 5(\sin^2 3\theta + \cos^2 3\theta)$

$\equiv 5$

c. $\frac{\sqrt{1-\cos^2 x}}{\cos x}$

$= \frac{\sin x}{\cos x}$

$\equiv \tan x$

d. $\sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta$

$\equiv (\sin^2\theta + \cos^2\theta)^2$

$\equiv 1^2$

$\equiv 1$

2. Prove that:

a. $(\sin\theta + \cos\theta)^2 \equiv 1 + 2\sin\theta\cos\theta$

$LHS \equiv \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$\equiv 1 + 2\sin\theta\cos\theta$

$\equiv RHS$

b. $\frac{1}{\cos\theta} - \cos\theta \equiv \sin\theta\tan\theta$

$LHS \equiv \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}$

$\equiv \frac{1-\cos^2\theta}{\cos\theta}$

$\equiv \frac{\sin^2\theta}{\cos\theta}$

$\equiv \sin\theta\frac{\sin\theta}{\cos\theta}$

$\equiv \sin\theta\tan\theta$

$\equiv RHS$

c. $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$

$LHS \equiv \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\equiv \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\equiv \frac{1}{\sin\theta\cos\theta}$

$\equiv RHS$

d. $(2\sin\theta - \cos\theta)^2 + (\sin\theta + 2\cos\theta)^2 \equiv 5$

$LHS \equiv 4\sin^2\theta - 4\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta + 4\sin\theta\cos\theta + 4\cos^2\theta$

$\equiv 5\sin^2\theta + 5\cos^2\theta$

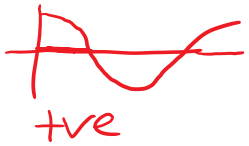
$\equiv 5(\sin^2\theta + \cos^2\theta)$

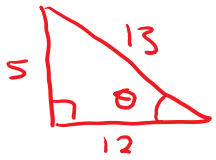
$\equiv 5$

$\equiv RHS$

3. Find the value of the following, without using a calculator:

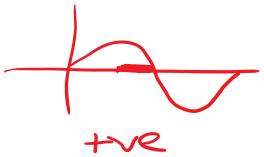
a. $\cos\theta$, given that $\tan\theta = \frac{5}{12}$ and θ is acute

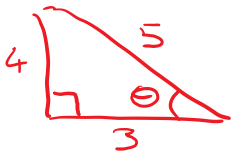




$\therefore \cos\theta = \frac{12}{13}$

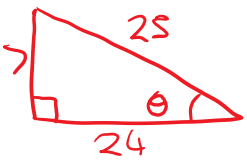
b. $\sin\theta$, given that $\cos\theta = -\frac{3}{5}$ and θ is obtuse





$\therefore \sin\theta = \frac{4}{5}$

c. $\tan\theta$, given that $\sin\theta = -\frac{7}{25}$ and $270° < \theta < 360°$



$\therefore \tan\theta = -\frac{7}{24}$