Factor Theorem and Algebraic Division GREEN

1. a) Divide $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$ b) Divide $x^3 - 5x^2 - 6x - 56$ by $(x - 7)$
   c) Divide $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$ d) Divide $-5x^3 - 27x^2 + 23x + 30$ by $(x + 6)$

2. a) Divide $3x^4 + 8x^3 - 11x^2 + 2x + 8$ by $(3x + 2)$
   b) Divide $21x^5 + 29x^4 - 10x^3 + 42x - 12$ by $(7x - 2)$
   c) Divide $2x^3 + 9x^2 + 25$ by $(x + 5)$

3. a) Find the remainder when $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$
   b) Find the remainder when $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x - 4)$

4. a) Show that $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$ and hence factorise the expression completely
   b) Show that $(x - 5)$ is a factor of $x^3 - 7x^2 + 2x + 40$ and hence factorise the expression completely

5. Fully factorise each cubic and hence sketch the graphs, showing all intercepts with axes
   a) $y = 2x^3 + 5x^2 - 4x - 3$ b) $y = 2x^3 - 17x^2 + 38x - 15$
   c) $y = 6x^3 + 11x^2 - 3x - 2$ d) $y = 4x^3 - 12x^2 - 7x + 30$

6. $f(x) = 3x^3 - 12x^2 + 6x - 24$
   a) Use the factor theorem to show that $(x - 4)$ is a factor of $f(x)$
   b) Hence show that 4 is the only real root of the equation $f(x) = 0$

7. $f(x) = 4x^3 + 4x^2 - 11x - 6$
   a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$
   b) Factorise $f(x)$ completely
   c) Write down the solutions of the equation $4x^3 + 4x^2 - 11x - 6 = 0$

8. a) Show that $(x - 2)$ is a factor of $9x^4 - 18x^3 - x^2 + 2x$
   b) Hence find four real solutions to the equation $9x^4 - 18x^3 - x^2 + 2x = 0$

Factor Theorem and Algebraic Division AMBER

1. a) Divide $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$
   b) Divide $x^3 - 5x^2 - 6x - 56$ by $(x - 7)$
   c) Divide $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$
   d) Divide $-5x^3 - 27x^2 + 23x + 30$ by $(x + 6)$

2. a) Divide $3x^4 + 8x^3 - 11x^2 + 2x + 8$ by $(3x + 2)$
   b) Divide $21x^5 + 29x^4 - 10x^3 + 42x - 12$ by $(7x - 2)$
   c) Divide $2x^3 + 9x^2 + 25$ by $(x + 5)$

Fill the gaps with zeros to help you divide

3. a) Find the remainder when $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$
   b) Find the remainder when $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x - 4)$

4. a) Show that $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$ and hence factorise the expression completely
   b) Show that $(x - 5)$ is a factor of $x^3 - 7x^2 + 2x + 40$ and hence factorise the expression completely

Substitute x = 1

5. Fully factorise each cubic and hence sketch the graphs, showing all intercepts with axes
   a) $y = 2x^3 + 5x^2 - 4x - 3$
   b) $y = 2x^3 - 17x^2 + 38x - 15$
   c) $y = 6x^3 + 11x^2 - 3x - 2$
   d) $y = 4x^3 - 12x^2 - 7x + 30$

6. $f(x) = 3x^3 - 12x^2 + 6x - 24$
   a) Use the factor theorem to show that $(x - 4)$ is a factor of $f(x)$
   b) Hence show that 4 is the only real root of the equation $f(x) = 0$

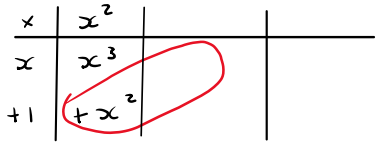
Use the discriminant

7. $f(x) = 4x^3 + 4x^2 - 11x - 6$
   a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$
   b) Factorise $f(x)$ completely
   c) Write down the solutions of the equation $4x^3 + 4x^2 - 11x - 6 = 0$

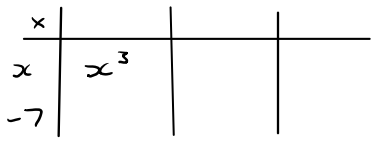
8. a) Show that $(x - 2)$ is a factor of $9x^4 - 18x^3 - x^2 + 2x$
   b) Hence find four real solutions to the equation $9x^4 - 18x^3 - x^2 + 2x = 0$

Factor Theorem and Algebraic Division RED

1. a) Divide $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$



b) Divide $x^3 - 5x^2 - 6x - 56$ by $(x - 7)$



c) Divide $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$

d) Divide $-5x^3 - 27x^2 + 23x + 30$ by $(x + 6)$

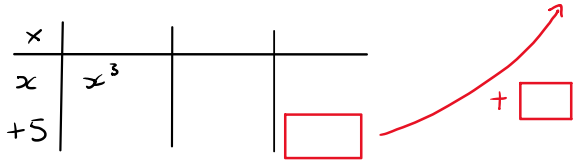
2. a) Divide $3x^4 + 8x^3 - 11x^2 + 2x + 8$ by $(3x + 2)$

Fill the gaps with zeros to help you divide

b) Divide $21x^5 + 29x^4 - 10x^3 + 0x^2 + 42x - 12$ by $(7x - 2)$

c) Divide $2x^3 + 9x^2 + 0x + 25$ by $(x + 5)$

3. a) Find the remainder when $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$



b) Find the remainder when $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x - 4)$

4. a) Show that $(x-1)$ is a factor of $x^3+6x^2+5x-12$ and hence factorise the expression completely

Substitute x = 1

$x = 1 \Rightarrow (1)^3 + 6(1)^2 + 5(1) - 12 =$

x			
x	x^3		
-1			

b) Show that $(x-5)$ is a factor of $x^3-7x^2+2x+40$ and hence factorise the expression completely

5. Fully factorise each cubic and hence sketch the graphs, showing all intercepts with axes

a) $y = 2x^3+5x^2-4x-3$

b) $y = 2x^3-17x^2+38x-15$

Find a factor first!

c) $y = 6x^3+11x^2-3x-2$

d) $y = 4x^3-12x^2-7x+30$

6. $f(x) = 3x^3 - 12x^2 + 6x - 24$

a) Use the factor theorem to show that $(x - 4)$ is a factor of $f(x)$

$f(4) =$

b) Hence show that 4 is the only real root of the equation $f(x) = 0$

Use the discriminant

×			
x	$3x^3$		
-4			

7. $f(x) = 4x^3 + 4x^2 - 11x - 6$

a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$

b) Factorise $f(x)$ completely

c) Write down the solutions of the equation $4x^3 + 4x^2 - 11x - 6 = 0$

8. a) Show that $(x - 2)$ is a factor of $9x^4 - 18x^3 - x^2 + 2x$

b) Hence find four real solutions to the equation $9x^4 - 18x^3 - x^2 + 2x = 0$

Factor Theorem and Algebraic Division ANSWERS

1. a) Divide $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$

×	x^2	$+5x$	$+3$
x	x^3	$5x^2$	$+3x$
$+1$	$+x^2$	$+5x$	$+3$

$= x^2 + 5x + 3$

b) Divide $x^3 - 5x^2 - 6x - 56$ by $(x - 7)$

×	x^2	$+2x$	$+8$
x	x^3	$+2x^2$	$+8x$
-7	$-7x^2$	$-14x$	-56

$= x^2 + 2x + 8$

c) Divide $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$

×	$4x^2$	$+x$	-5
x	$4x^3$	$+x^2$	$-5x$
$+2$	$+8x^2$	$+2x$	-10

$= 4x^2 + x - 5$

d) Divide $-5x^3 - 27x^2 + 23x + 30$ by $(x + 6)$

×	$-5x^2$	$+3x$	$+5$
x	$-5x^3$	$+3x^2$	$+5x$
$+6$	$-30x^2$	$+18x$	$+30$

$= -5x^2 + 3x + 5$

2. a) Divide $3x^4 + 8x^3 - 11x^2 + 2x + 8$ by $(3x + 2)$

×	x^3	$+2x^2$	$-5x$	$+4$
$3x$	$3x^4$	$+6x^3$	$-15x^2$	$+12x$
$+2$	$+2x^3$	$+4x^2$	$-10x$	$+8$

$= x^3 + 2x^2 - 5x + 4$

b) Divide $21x^5 + 29x^4 - 10x^3 + 0x^2 + 42x - 12$ by $(7x - 2)$

×	$3x^4$	$+5x^3$	$+0x^2$	$+0x$	$+6$
$7x$	$21x^5$	$+35x^4$	$+0x^3$	$+0x^2$	$+42x$
-2	$-6x^4$	$-10x^3$	$+0x^2$	$+0x$	-12

$= 3x^4 + 5x^3 + 6$

c) Divide $2x^3 + 9x^2 + 0x + 25$ by $(x + 5)$

×	$2x^2$	$-x$	$+5$
x	$2x^3$	$-x^2$	$+5x$
$+5$	$+10x^2$	$-5x$	$+25$

$= 2x^2 - x + 5$

3. a) Find the remainder when $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$

×	x^2	$-x$	$+2$
x	x^3	$-x^2$	$+2x$
$+5$	$+5x^2$	$-5x$	$+10$

$+ \boxed{-8}$

Remainder $= -8$

b) Find the remainder when $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x - 4)$

×	$-2x^2$	$-5x$	-8
x	$-2x^3$	$-5x^2$	$-8x$
-4	$+8x^2$	$+20x$	$+32$

Remainder $= -12$

4. a) Show that $(x-1)$ is a factor of $x^3+6x^2+5x-12$ and hence factorise the expression completely

$x=1 \Rightarrow (1)^3+6(1)^2+5(1)-12 = 1+6+5-12=0$

$\therefore (x-1)$ is a factor

$\times$	x^2	$+7x$	$+12$
x	x^3	$+7x^2$	$+12x$
-1	$-x^2$	$-7x$	-12

$x^3+6x^2+5x-12 = (x-1)(x^2+7x+12) = (x-1)(x+3)(x+4)$

b) Show that $(x-5)$ is a factor of $x^3-7x^2+2x+40$ and hence factorise the expression completely

$x=5 \Rightarrow (5)^3-7(5)^2+2(5)+40 = 125-175+10+40=0$

$\therefore (x-5)$ is a factor.

$\times$	x^2	$-2x$	-8
x	x^3	$-2x^2$	$-8x$
-5	$-5x^2$	$+10x$	$+40$

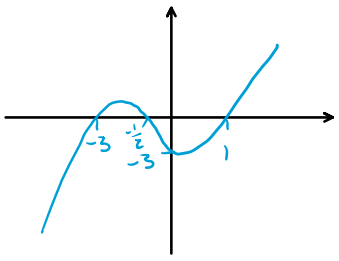
$x^3-7x^2+2x+40 = (x-5)(x^2-2x-8) = (x-5)(x-4)(x+2)$

5. Fully factorise each cubic and hence sketch the graphs, showing all intercepts with axes

a) $y=2x^3+5x^2-4x-3$

$\times$	$2x^2$	$+7x$	$+3$
x	$2x^3$	$+7x^2$	$+3x$
-1	$-2x^2$	$-7x$	-3

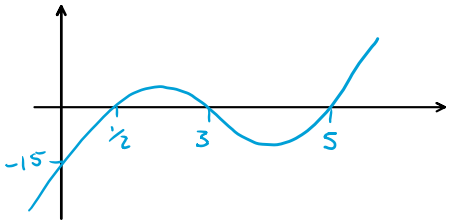
$y=(x-1)(x+3)(2x+1)$



b) $y=2x^3-17x^2+38x-15$

$\times$	$2x^2$	$-11x$	$+5$
x	$2x^3$	$-11x^2$	$+5x$
-3	$-6x^2$	$+33x$	-15

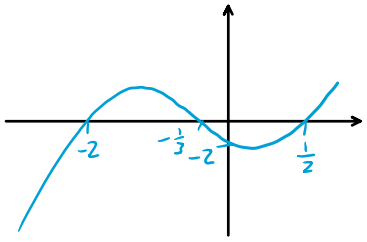
$y=(x-3)(2x-1)(x-5)$



c) $y=6x^3+11x^2-3x-2$

$\times$	$6x^2$	$-x$	-1
x	$6x^3$	$-x^2$	$-x$
$+2$	$+12x^2$	$-2x$	-2

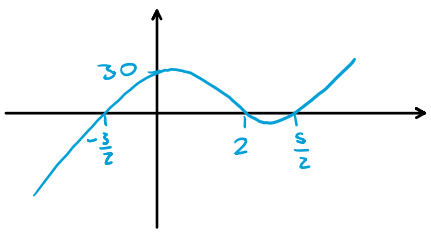
$y=(x+2)(2x-1)(3x+1)$



d) $y=4x^3-12x^2-7x+30$

$\times$	$4x^2$	$-4x$	-15
x	$4x^3$	$-4x^2$	$-15x$
-2	$-8x^2$	$+8x$	$+30$

$y=(x-2)(2x-5)(2x+3)$



6. $f(x) = 3x^3 - 12x^2 + 6x - 24$

a) Use the factor theorem to show that $(x - 4)$ is a factor of $f(x)$

$f(4) = 3(4)^3 - 12(4)^2 + 6(4) - 24 = 192 - 192 + 24 - 24 = 0$

b) Hence show that 4 is the only real root of the equation $f(x) = 0$

×	$3x^2$	$+0x$	$+6$
x	$3x^3$	$0x^2$	$+6x$
-4	$-12x^2$	$0x$	-24

$b^2 - 4ac = 0^2 - 4 \times 3 \times 6 = -72 < 0$

Discriminant < 0 $\therefore$ no other real solutions of $f(x)$

7. $f(x) = 4x^3 + 4x^2 - 11x - 6$

a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$

$f(-2) = 4(-2)^3 + 4(-2)^2 - 11(-2) - 6 = -32 + 16 + 22 - 6 = 0$

b) Factorise $f(x)$ completely

×	$4x^2$	$-4x$	-3
x	$4x^3$	$-4x^2$	$-3x$
$+2$	$+8x^2$	$-8x$	-6

$f(x) = (x+2)(4x^2 - 4x - 3) = (x+2)(2x+1)(2x-3)$

c) Write down the solutions of the equation $4x^3 + 4x^2 - 11x - 6 = 0$

$x = -2, -\frac{1}{2}, \frac{3}{2}$

8. a) Show that $(x - 2)$ is a factor of $9x^4 - 18x^3 - x^2 + 2x$

$x = 2 \Rightarrow 9(2)^4 - 18(2)^3 - (2)^2 + 2(2) = 144 - 144 - 4 + 4 = 0$

b) Hence find four real solutions to the equation $9x^4 - 18x^3 - x^2 + 2x = 0$

×	$9x^3$	$+0x^2$	$-x$	$+0$
x	$9x^4$	$+0x^3$	$-x^2$	$+0x$
-2	$-18x^3$	$+0x^2$	$+2x$	$+0$

$9x^4 - 18x^3 - x^2 + 2x = x(x-2)(9x^2 - 1)$

$= x(x-2)(3x+1)(3x-1)$