Geometrical Reasoning GREEN

1.

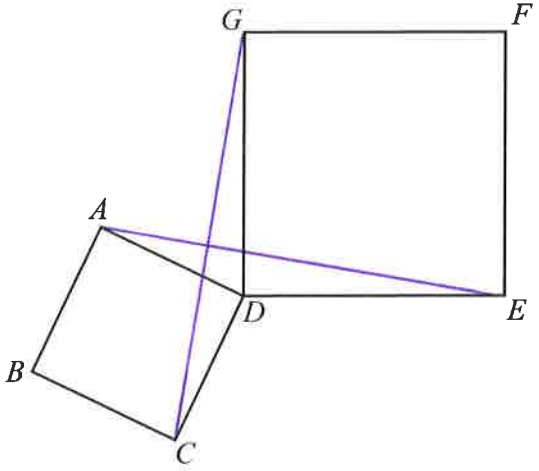


Diagram **NOT** accurately drawn

ABCD and *DEFG* are squares.

Prove that triangle *CDG* and triangle *ADE* are congruent.

CD = AD (square ABCD has equal side lengths)

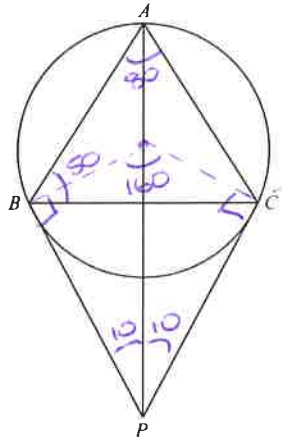
DG = DE (square DEFG has equal side lengths)

CDG = ADE = 90° + ADG (squares have all angles 90°)

SAS proves triangles are congruent.

(Total 3 marks)

2.



A, *B* and *C* are three points on the circumference of a circle.
Angle *ABC* = Angle *ACB*.
PB and *PC* are tangents to the circle from the point *P*.

(a) Prove that triangle *APB* and triangle *APC* are congruent.

PB = PC (tangents to same point)

AB = AC (isosceles triangle).

PBA = PCA (ABC isosceles, BCP isosceles)

SAS proves triangles are congruent.

(3)

Angle *BPA* = 10°.

(b) Find the size of angle *ABC*.

10 + 10 = 20°

~~180 - (2 × 90)~~

360 − (2 × 90 + 20) = 160°

160 ÷ 2 = 80°

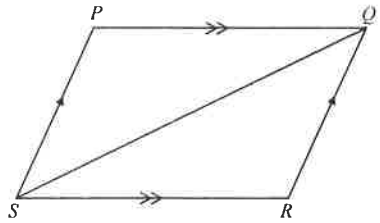
$\frac{180 - 80°}{2} = 50°$

50 °

(4)

(Total 7 marks)

3. *PQRS* is a quadrilateral.



PQ is parallel to *SR*.
SP is parallel to *RQ*.

(a) Prove that triangle *PQS* is congruent to triangle *RSQ*.

QS = QS (same length)
PQ = SR (parallelogram)
PS = QR (parallelogram)
SSS proves triangles are congruent.

(3)

(b) In quadrilateral *PQRS*, angle *SPQ* is obtuse.
Explain why *PQRS* cannot be a cyclic quadrilateral.

SPQ = SRQ (opposites in parallelogram equal)
SPQ + SRQ > 180°
Opposite angles in cyclic quad. add up to 180°.

(2)
(Total 5 marks)

4.

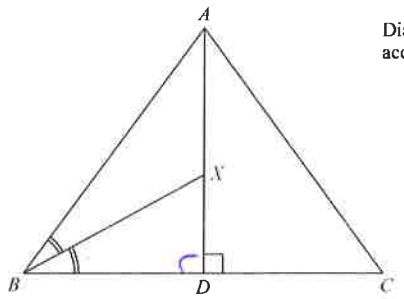


Diagram **NOT** accurately drawn

ABC is an equilateral triangle.
AD is the perpendicular bisector of *BC*.
BX is the angle bisector of angle *ABC*.

(a) Show that triangle *BXD* is similar to triangle *ACD*.

BDX = ADC (right-angle)
ACD = 60° (equilateral)
DAC = 30° (180 - 60 + 90)
DBX = 30° (60 ÷ 2)
DXB = 60° (180 - 30 + 90)
AAA proves triangles are similar.

(2)

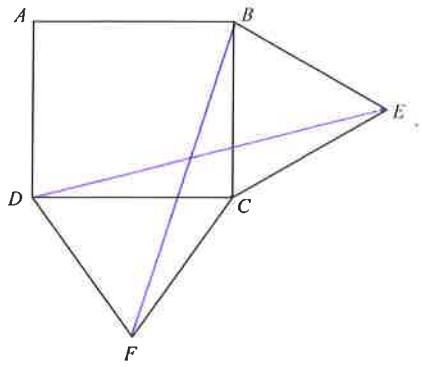
In triangle *ACD*,
AC = 2 cm,
AD = $\sqrt{3}$ cm.

(b) Show that $XD = \frac{1}{\sqrt{3}}$ cm.

Scale factor = ~~vertical~~ $\sqrt{3} \div 1 = \sqrt{3}$
$XD = DC \div \sqrt{3} = \frac{1}{\sqrt{3}}$.

(3)
(Total 5 marks)

5. Diagram NOT accurately drawn



ABCD is a square.
BEC and *DCF* are equilateral triangles.

(a) Prove that triangle *ECD* is congruent to triangle *BCF*.

FC = CE (equilateral triangles same length as square)
CD = BC (square sides equal)
FCB = DCE = 90 + 60
SAS proves triangles are congruent.

(3)

G is the point such that *BEGF* is a parallelogram.

(b) Prove that *ED* = *EG*

DE = BF (congruent triangles)
BF = EG (parallelogram)
∴ ED = EG.

(2)
(Total 5 marks)

6.

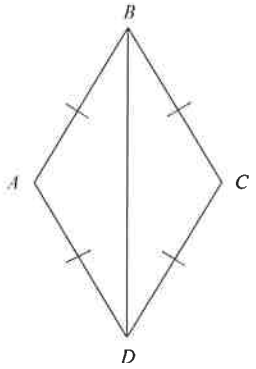


Diagram NOT accurately drawn

In the diagram, *AB* = *BC* = *CD* = *DA*.

Prove that triangle *ADB* is congruent to triangle *CDB*.

AD = CD (question)
AB = BC (question)
BD shared by both triangles.
SSS proves triangles are congruent

(Total 3 marks)

7.

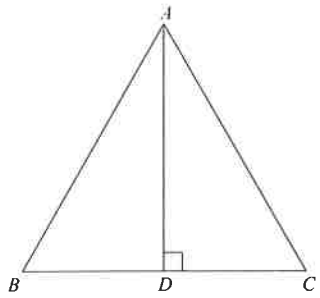


Diagram **NOT** accurately drawn

ABC is an equilateral triangle.
D lies on *BC*.
AD is perpendicular to *BC*.

(a) Prove that triangle *ADC* is congruent to triangle *ADB*.

D is midpoint of BC
∴ BD = DC
BDA = ADC = 90°
AD shared by both triangles
SAS proves triangles are congruent.

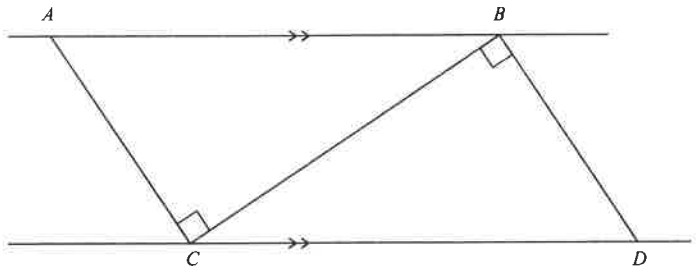
(3)

(b) Hence, prove that $BD = \frac{1}{2}AB$.

AB = BC.
$\frac{1}{2}BC = \frac{1}{2}AB$
$BD = \frac{1}{2}AB$.

(2)
(Total 5 marks)

8.



AB is parallel to *CD*.
Angle *ACB* = angle *CBD* = 90°.

Prove that triangle *ABC* is congruent to triangle *DCB*.

AC and BD parallel
ABC = BCD (alternate angles)
CB shared by both triangles.
ACB = CBD = 90°
ASA proves triangles are congruent.

(Total 3 marks)