Tangents, Normals and Stationary Points GREEN

1. Find the equation of the tangent to the curve $y = x^2 - 7x + 10$ at the point $(2, 0)$.

2. Find the equation of the normal to the curve $y = x^2 - 5x$ at the point $(6, 6)$.

3. Find the coordinates of the points where the gradient is zero on the given curve. Establish whether these points are local minimum points, local maximum points or points of inflection in each case.

a. $y = 2x^3 + 5x^2 - 4x + 3$

b. $y = x^3 + 6x^2 + 9x + 1$

Tangents, Normals and Stationary Points AMBER

1. Find the equation of the tangent to the curve $y = x^2 - 7x + 10$ at the point $(2, 0)$.

1. Differentiate
2. Calculate gradient
3. Use $y - y_1 = m(x - x_1)$

2. Find the equation of the normal to the curve $y = x^2 - 5x$ at the point $(6, 6)$.

1. Differentiate
2. Calculate gradient
3. Calculate perpendicular gradient
4. Use $y - y_1 = m(x - x_1)$

3. Find the coordinates of the points where the gradient is zero on the given curve. Establish whether these points are local minimum points, local maximum points or points of inflection in each case.

1. Differentiate
2. Solve with $\frac{dy}{dx} = 0$
3. Calculate $\frac{d^2y}{dx^2}$
4. Substitute x values
5. Determine nature of stationary points

a. $y = 2x^3 + 5x^2 - 4x + 3$

b. $y = x^3 + 6x^2 + 9x + 1$

Tangents, Normals and Stationary Points RED

1. Find the equation of the tangent to the curve $y = x^2 - 7x + 10$ at the point $(2, 0)$.

$\frac{dy}{dx} =$

1. Differentiate
2. Calculate gradient
3. Use $y - y_1 = m(x - x_1)$

2. Find the equation of the normal to the curve $y = x^2 - 5x$ at the point $(6, 6)$.

1. Differentiate
2. Calculate gradient
3. Calculate perpendicular gradient
4. Use $y - y_1 = m(x - x_1)$

3. Find the coordinates of the points where the gradient is zero on the given curve. Establish whether these points are local minimum points, local maximum points or points of inflection in each case.

1. Differentiate
2. Solve with $\frac{dy}{dx} = 0$
3. Calculate $\frac{d^2y}{dx^2}$
4. Substitute x values
5. Determine nature of stationary points

a. $y = 2x^3 + 5x^2 - 4x + 3$

$\frac{dy}{dx} = 6x^2 + 10x - 4 = 0$ ← factorise this!

$\frac{d^2y}{dx^2} =$ (substitute x values into this!)

b. $y = x^3 + 6x^2 + 9x + 1$

1. Differentiate
2. Solve with $\frac{dy}{dx} = 0$
3. Calculate $\frac{d^2y}{dx^2}$
4. Substitute x values
5. Determine nature of stationary points

Tangents, Normals and Stationary Points ANSWERS

1. Find the equation of the tangent to the curve $y = x^2 - 7x + 10$ at the point $(2, 0)$.

$\frac{dy}{dx} = 2x - 7$

At $x = 2$, $\frac{dy}{dx} = 2 \times 2 - 7 = -3$

$y - 0 = -3(x - 2) \Rightarrow y = -3x + 6$

2. Find the equation of the normal to the curve $y = x^2 - 5x$ at the point $(6, 6)$.

$\frac{dy}{dx} = 2x - 5$

At $x = 6$, $\frac{dy}{dx} = 2 \times 6 - 5 = 7$

$y - 6 = 7(x - 6) \Rightarrow y = 7x - 36$

3. Find the coordinates of the points where the gradient is zero on the given curve. Establish whether these points are local minimum points, local maximum points or points of inflection in each case.

a. $y = 2x^3 + 5x^2 - 4x + 3$

$\frac{dy}{dx} = 6x^2 + 10x - 4 \Rightarrow 0 = 3x^2 + 5x - 2$

$0 = (3x-1)(x+2)$

$x = \frac{1}{3}$ and -2

$\frac{d^2y}{dx^2} = 12x + 10$

When $x = \frac{1}{3}$, $y = \frac{62}{27}$, $\frac{d^2y}{dx^2} = 14 > 0$

$\therefore \left(\frac{1}{3}, \frac{62}{27}\right)$ is a local minimum

When $x = -2$, $y = 15$, $\frac{d^2y}{dx^2} = -14 < 0$

$\therefore (-2, 15)$ is a local maximum.

b. $y = x^3 + 6x^2 + 9x + 1$

$\frac{dy}{dx} = 3x^2 + 12x + 9 \Rightarrow x^2 + 4x + 3 = 0$

$(x+1)(x+3) = 0$

$x = -1$ and -3

$\frac{d^2y}{dx^2} = 6x + 12$

When $x = -1$, $y = -3$, $\frac{d^2y}{dx^2} = 6 > 0$

$\therefore (-1, -3)$ is a local minimum

When $x = -3$, $y = 1$, $\frac{d^2y}{dx^2} = -6 < 0$

$\therefore (-3, 1)$ is a local maximum