

Maths Problem Solving Starters

Levels 4 – 6

Name: Worked solutions.

Class: _____

Teacher: _____

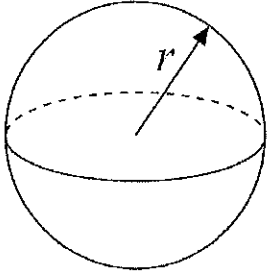
Remember:

- You will need: pen, pencil, rubber and a ruler.
- Check your work carefully.
- Show all of your working out, with clear steps.

Formulae Sheet

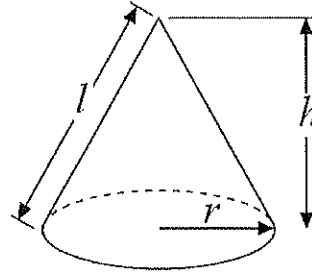
Perimeter, area, surface area and volume formulae

Sphere



$$\text{Volume} = \frac{4}{3} \pi r^3$$
$$\text{Surface Area} = 4\pi r^2$$

Cone



$$\text{Volume} = \frac{1}{3} \pi r^2 h$$
$$\text{Curved Surface Area} = \pi r l$$

1. Fill in the gaps:

$$a) (x + 2)(x + \boxed{3}) = x^2 + \boxed{5}x + 6$$

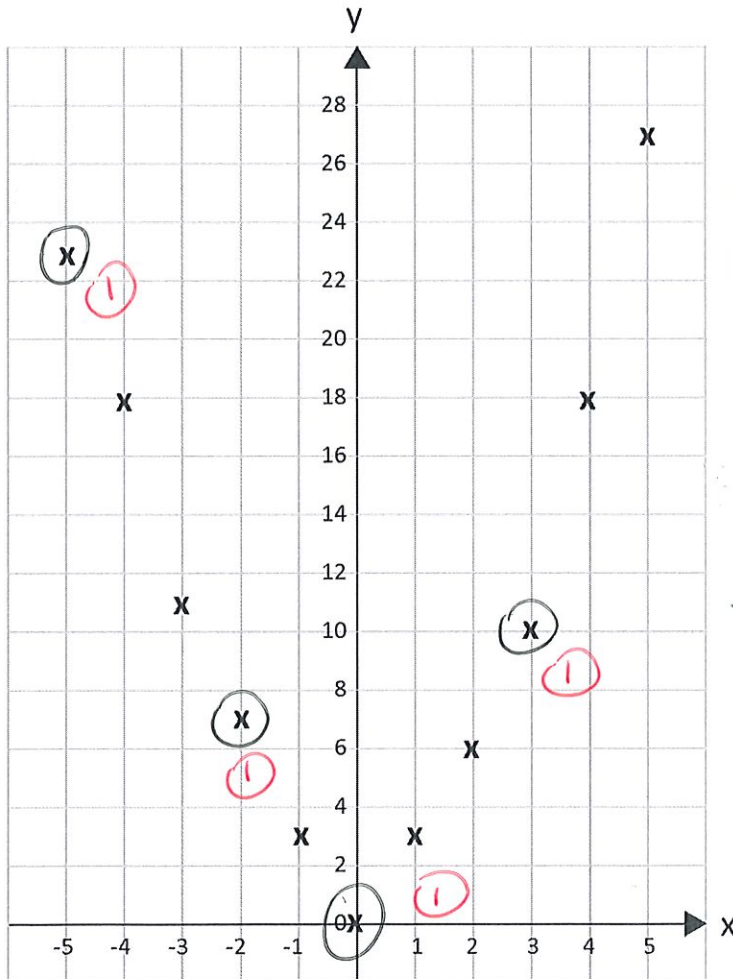
$$b) (x - \boxed{3})(x + 8) = x^2 + 5x - \boxed{24}$$

$$(x + 2)(x + a) = x^2 + 2x + ax + 2a = x^2 + (2 + a)x + 2a$$

$$(x - b)(x + 8) = x^2 + (8 - b)x - 8b$$

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2. Enzo makes a table of values and plots the graph of $y = x^2 + 2$. Which points on the graph are incorrect?



$$\begin{aligned} (-5)^2 + 2 &= 27 \\ (-4)^2 + 2 &= 18 \\ (-3)^2 + 2 &= 11 \\ (-2)^2 + 2 &= 6 \\ (-1)^2 + 2 &= 3 \\ 0^2 + 2 &= 2 \\ 1^2 + 2 &= 3 \\ 2^2 + 2 &= 6 \\ 3^2 + 2 &= 11 \\ 4^2 + 2 &= 18 \\ 5^2 + 2 &= 27 \end{aligned}$$

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3.

A population of ants increases at a rate of 30% per day. At the end of one week there are 3500 insects. How many insects were there at the beginning of the week?



$$x \times 1.3^7 = 3500$$

$$x = 3500 \div 1.3^7$$

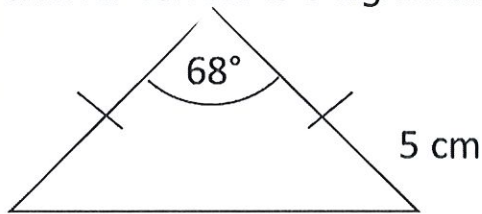
$$= 557.78\dots$$

558

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4.

Work out the area of this isosceles triangle. Give your answer correct to 3 significant figures.



$$\frac{1}{2} \times 5 \times 5 \times \sin 68 = 11.589\dots$$

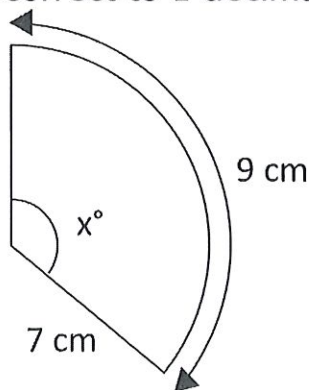
11.6

cm²

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5.

Find the angle of this sector. Give your answer correct to 1 decimal place.



$$14\pi \times \frac{x}{360} = 9$$

$$\frac{x}{360} = \frac{9}{14\pi}$$

$$x = \frac{9 \times 360}{14\pi}$$

$$= 73.666\dots$$

73.7 °

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6. The pressure, P , of water on an object (in bars) is directly proportional to its depth, d (in metres). When the object is at a depth of 8 metres, the pressure on the object is 0.8 bars. A diver's watch has been guaranteed to work at pressures up to 8.5 bars. The diver takes the watch down to 75 m. Will the watch still work?

$$P \propto d \quad (1)$$

$$P = kd \quad (1)$$

$$0.8 = 8k \quad (1)$$

$$0.1 = k \quad (1)$$

$$P = 0.1d \quad (1)$$

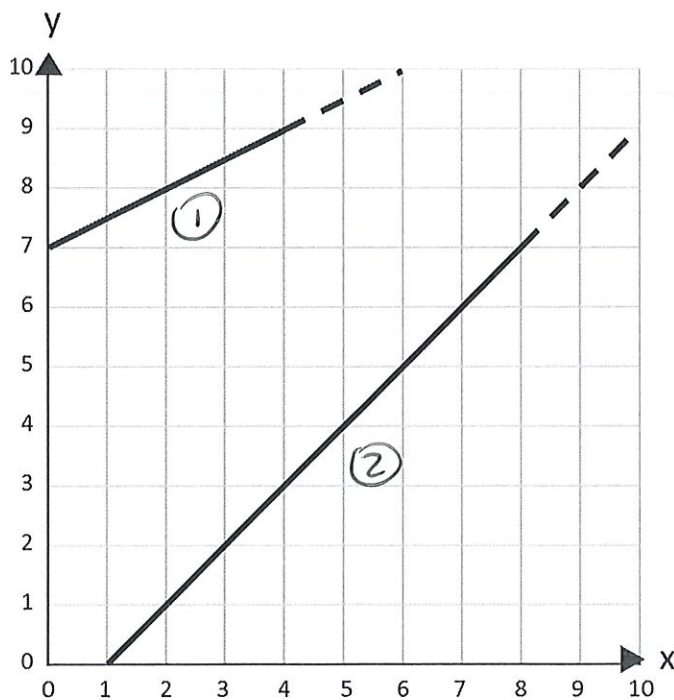
$$P = 0.1 \times 75 \quad (1)$$

$$= 7.5 \text{ bars.} \quad (1)$$

Yes the watch will still work. (1)

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7. Find the coordinates of the point where these two lines meet if they are extended.



$$(1) : y = \frac{1}{2}x + 7 \quad (1)$$

$$(2) : y = x - 1 \quad (1)$$

$$\frac{1}{2}x + 7 = x - 1 \quad (1)$$

$$x + 14 = 2x - 2 \quad (1)$$

$$16 = x \quad (1)$$

$$y = x - 1 = 16 - 1 = 15 \quad (1) \quad (1) \quad (16, 15)$$

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8.

The rule for a sequence of number pairs is:

(first number, last number) →**(first number + last number, first number – last number)**

eg $(5, 3) \rightarrow (8, 2)$

\swarrow $5+3$
 \searrow $5-3$

Here is part of a sequence that follows this rule. Write in the missing number pairs.

$$\left(\frac{0.5}{c}, \frac{-1}{d}\right) \left(\frac{-0.5}{a}, \frac{1.5}{b}\right) (1, 2) (3, -1) (2, 4) \left(\frac{6}{e}, \frac{-2}{f}\right)$$

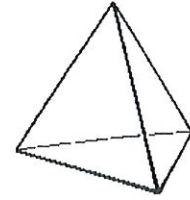
$$\begin{aligned} a+b &= 1 \\ -a-b &= 2 \quad \textcircled{1} \\ \hline 2b &= 3 \\ b &= 1.5 \quad \textcircled{1} \\ a &= -0.5 \quad \textcircled{1} \end{aligned}$$

$$\begin{aligned} e: 2+4 &= 6 \quad \textcircled{1} \\ f: 2-4 &= -2 \quad \textcircled{1} \end{aligned}$$

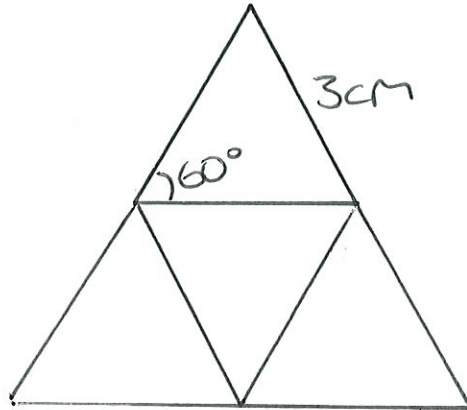
$$\begin{aligned} c+d &= -0.5 \\ -c-d &= 1.5 \\ \hline 2d &= -2 \\ d &= -1 \quad \textcircled{1} \\ c &= 0.5 \quad \textcircled{1} \end{aligned}$$

9.

This chocolate box is in the shape of a tetrahedron. Each face is an equilateral triangle with side length 24 cm. Construct an accurate net for the box. Use a scale of 1 cm to 8 cm.



$24 \div 8 = 3 \text{ cm sides. } \textcircled{1}$

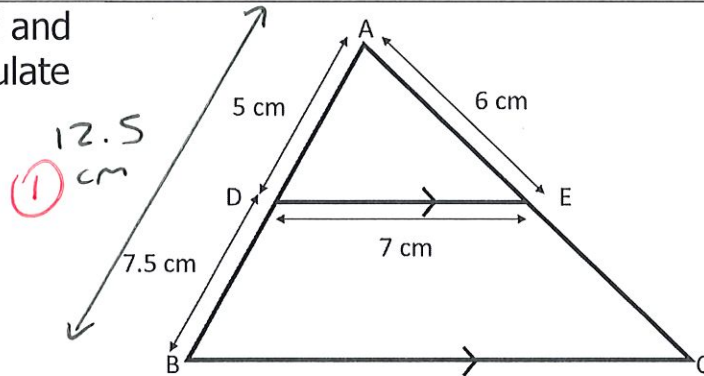


- $\textcircled{1}$ per incorrect angle/side.

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10.

ABC is a triangle. DE and BC are parallel. Calculate the perimeter of trapezium DBCE.



Scale factor : $12.5 \div 5 = 2.5 \textcircled{1}$

BC : $7 \times 2.5 = 17.5 \text{ cm } \textcircled{1}$

EC : $6 \times 2.5 - 6 = 9 \text{ cm } \textcircled{1}$

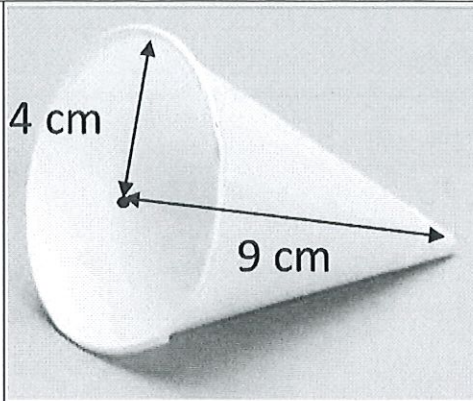
Perimeter : $7.5 + 7 + 17.5 + 9 = 41 \textcircled{1}$

$\textcircled{1}$

41 cm

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11.



Work out the area of card needed to make this disposable cup. Give your answer correct to 1 decimal place.



$$\text{Curved surface area} = \pi r l$$

$$l = \sqrt{4^2 + 9^2} = 9.848 \dots$$

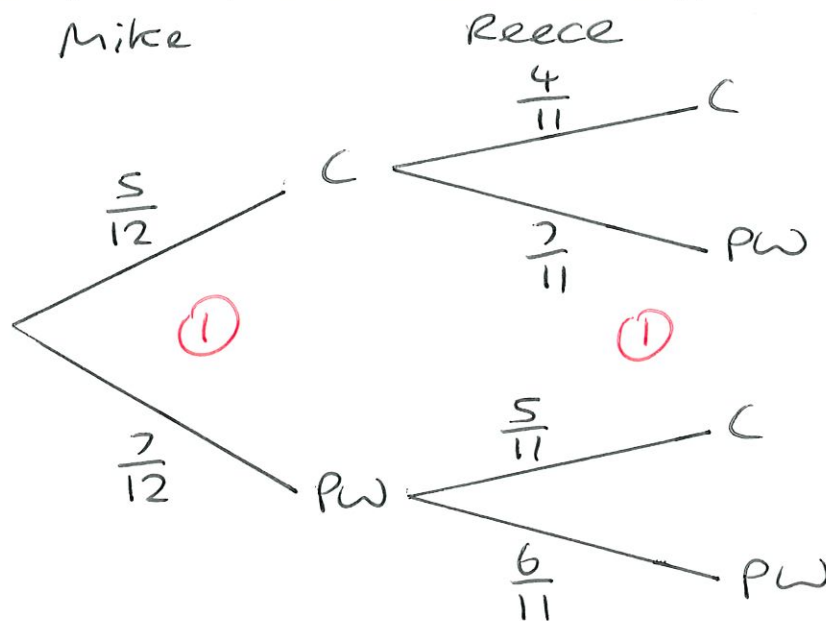
$$\text{Card needed} : \pi \times 4 \times 9.848 \dots = 123.76 \dots$$

$$\underline{123.8} \text{ cm}^2$$

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12.

There are 5 boxes of cornflakes and 7 boxes of puffed wheat. Mike and Reece both choose a box at random. Work out the probability that they do not choose the same type of cereal.



$$P(\text{C AND PW OR PW AND C})$$

$$= \frac{5}{12} \times \frac{7}{11} + \frac{7}{12} \times \frac{5}{11}$$

$$= \frac{70}{132}$$

$$\underline{\underline{\frac{70}{132}}}$$

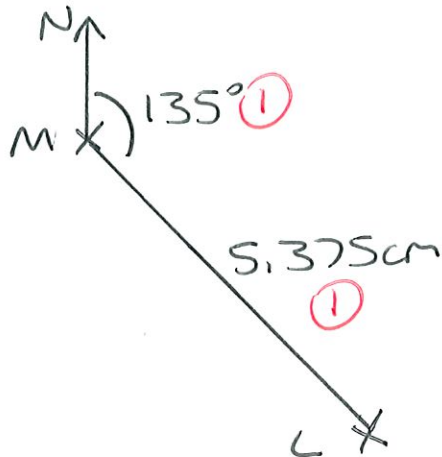
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13.

The distance between Manchester airport and Luton airport is 215 km. The bearing of Luton airport from Manchester airport is 135° . Make an accurate scale map of their locations using a scale of 1 cm to 40 km.



$$215 \div 4 = 5.375 \text{ cm} \text{ (1)}$$



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14.

A telephone company charges £ x per month for a basic line rental and then £ y per 100 minutes. Justin pays £18 for 200 minutes. Teresa pays £21 for 300 minutes. Work out the cost of the monthly rental.

$$\begin{array}{r} x + 2y = 18 \text{ (1)} \\ - x + 3y = 21 \text{ (1)} \\ \hline \end{array}$$

$$\begin{array}{r} -y = -3 \\ y = 3 \text{ (1)} \end{array}$$

$$\begin{array}{r} x + 2y = 18 \text{ (1)} \\ x + 6 = 18 \\ x = 12 \text{ (1)} \end{array}$$

£ 12

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15. Find a quadratic equation that has solutions $x = 0$ and $x = 5$
Give your answer without brackets.

$$(x + 0)(x - 5) = 0$$

$$x(x - 5) = 0 \quad (1)$$

$$x^2 - 5x = 0$$

$$\underline{x^2 - 5x = 0}$$

- Find a quadratic equation that has two solutions $x = 7$
Give your answer without brackets.

$$(x - 7)(x - 7) = 0 \quad (1)$$

$$x^2 - 7x - 7x + 49 = 0$$

$$\underline{x^2 - 14x + 49 = 0} \quad (1)$$

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16. A naturalist captures 30 bats in a cave and tags them. There are approximately 600 bats in the cave. The naturalist returns a month later and captures 40 bats. How many bats would he expect to be tagged?

$$\frac{30}{600} \times 40 = \frac{1}{20} \text{ of } 40 = 2$$

$$\underline{2} \quad (1)$$

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17. One sheet of paper is 9×10^{-3} cm thick. Mark wants to put 500 sheets of paper in the paper tray of his printer. The paper tray is 4 cm deep. Is the paper tray deep enough for 500 sheets of paper?

$$500 = 5 \times 10^2 \quad (1)$$

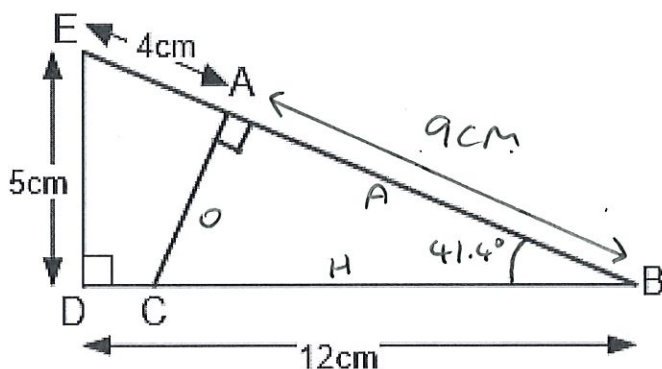
$$9 \times 10^{-3} \times 5 \times 10^2 = 45 \times 10^{-1} \quad (1)$$

$$= 4.5 \text{ cm} \quad (1)$$

No the tray is not deep enough. (1)

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18. The diagram shows two right-angled triangles ABC and DEB. Find the length of the line AC.



$$EB: \sqrt{12^2 + 5^2} = 13 \text{ cm} \quad (1)$$

$$AB: 13 - 4 = 9 \text{ cm} \quad (1)$$

$$ABC: \cos^{-1}(9 \div 12) = 41.4^\circ \quad (1)$$

$$AC: \tan 41.4 \times 9 = 7.937... \quad (1)$$

7.9 cm

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19. An aerial photograph shows a campsite with a swimming pool. In the photograph, the pool measures 5 cm x 2 cm. The real pool is 25 m long. How wide is it?

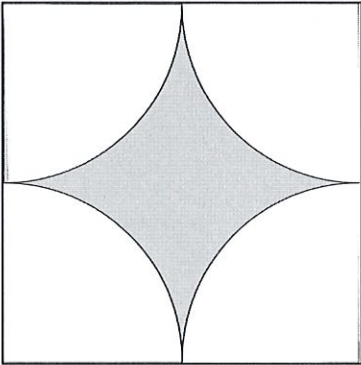

Scale factor : $2500 \div 5 = 500$ ①

$2 \times 500 = 1000 \text{ cm}$ ①

$= 10 \text{ m}$ ①

10 m

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20.  Four quarter circles are cut from a 10 cm square. Work out the shaded area and give your answer correct to 2 decimal places. 

$10^2 = 100 \text{ cm}^2$ ① (total area).

$4 \times \frac{1}{4} = 1$ whole.

$\pi \times 5^2 = 78.539 \dots \text{ cm}^2$ ①

$100 - 78.539 \dots = 21.460 \dots$

21.46 cm^2 ①

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21. Will says that $25^{-1/2} \times 64^{2/3} = 80$. Show that Will is wrong.

$25^{-1/2} = \frac{1}{\sqrt{25}} = \frac{1}{5}$ ①

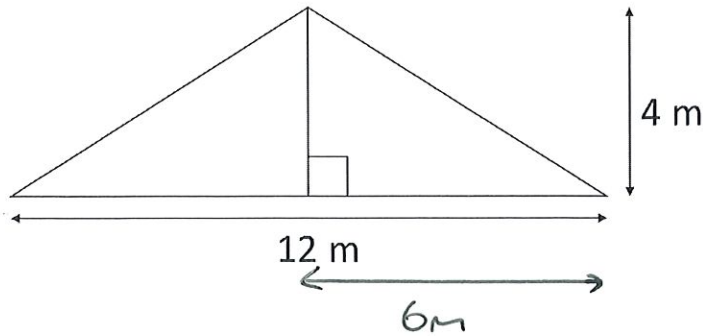
$64^{2/3} = \sqrt[3]{64}^2 = 4^2 = 16$ ①

$\frac{1}{5} \times 16 = 3.2$ ①

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22.

A roof truss is made of wood. The vertical support bisects the horizontal span. Work out the total length of wood needed to make the truss. Give your answer correct to 1 decimal place.



$$\sqrt{6^2 + 4^2} = 7.211... \quad (1)$$

$$12 + 4 + 2 \times 7.211... = 30.422... \quad (1)$$

$$\underline{\quad 30.4 \quad} \text{ m} \quad (1)$$

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23.

Two meals and a bottle of wine cost £36. The bottle of wine costs £3 more than a meal. How much do each cost?

$$2M + W = 36 \quad (1) \quad W = M + 3 \quad (1)$$

Substitute...

$$2M + M + 3 = 36 \quad (1)$$

$$3M + 3 = 36$$

$$3M = 33$$

$$M = 11 \quad (1)$$

$$W = M + 3 = 11 + 3 = 14 \quad (1)$$

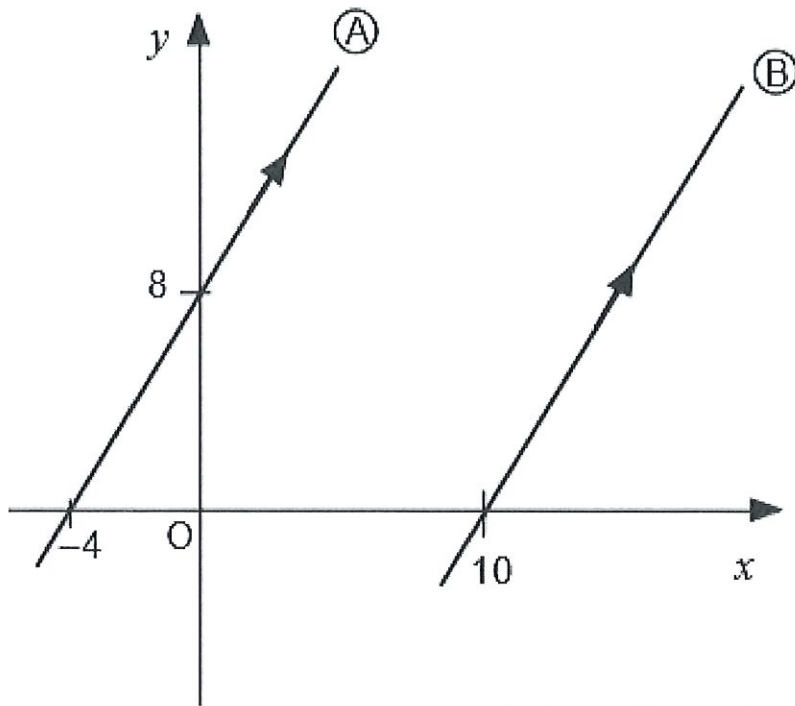
Wine: £ 14

Meal: £ 11

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24.

The lines A and B are parallel. What is the equation of line B?



not drawn to scale

$$A: y = 2x + 8$$

$$B: y = 2x + c \quad (\text{parallel } \therefore \text{ same gradient})$$

Substitute (10, 0)

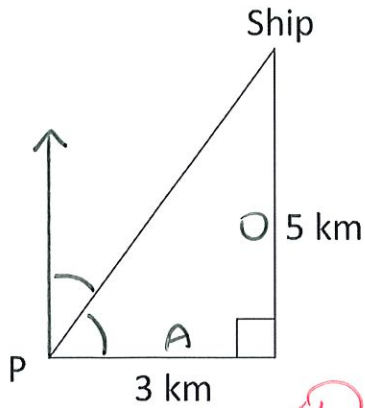
$$0 = 2 \times 10 + c$$

$$-20 = c$$

$$y = 2x - 20$$

$$\underline{y = 2x - 20}$$

25.



From P, a ship sails 3 km East and 5 km North to its destination. A helicopter flies from P directly to the ship. On what bearing from P should the helicopter fly? Give your answer correct to the nearest degree.

$$\tan^{-1}(5 \div 3) = 59.036\dots$$

$$90 - 59.036\dots = 30.963\dots$$

031°

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26.

In a circuit, the resistance, R ohms, is inversely proportional to the current, I amps. When the resistance is 12 ohms, the current in the circuit is 8 amps. Find the current when the resistance in the circuit is 6.4 ohms.

$$R \propto \frac{1}{I}$$

$$R = \frac{k}{I}$$

$$12 = \frac{k}{8}$$

$$96 = k$$

$$R = \frac{96}{I}$$

$$6.4 = \frac{96}{I}$$

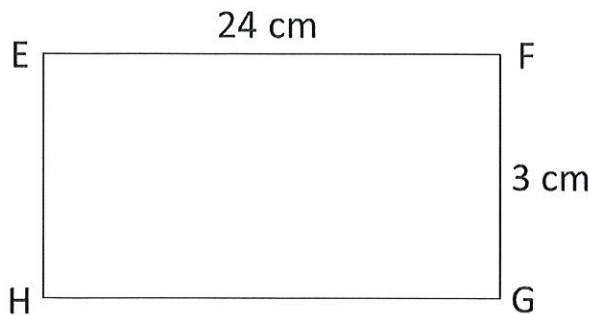
$$I = \frac{96}{6.4} = 15$$

15 amps

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27.

The diagram shows a rectangle EFGH. Length EF is 24 cm. width FG is 3cm. The length of the rectangle decreases by 40% and the width increases by 30%. What is the overall percentage change to the area of the rectangle?



Current area:
 $24 \times 3 = 72 \text{ cm}^2$
 (1)

$0.6 \times 24 = 14.4 \text{ cm}$ (1)

$1.3 \times 3 = 3.9 \text{ cm}$ (1)

New area: $14.4 \times 3.9 = 56.16$ (1)

$72 - 56.16 = 15.84$ (1)

$\frac{15.84}{72} \times 100 = 22\%$
 (1)

(1) 22 % (1) decrease