

Logarithms Exam Questions

1. a) $\log_a x = 3 \log_a 6 - \log_a 8$
 $= \log_a 6^3 - \log_a 8$
 $= \log_a \frac{6^3}{8}$
 $= \log_a \frac{216}{8}$
 $= \log_a 27$

$$8 \overline{) 216}$$

b) i) $\log_4 1 = 0$

ii) $\log_4 4 = 1$

iii) $\log_4 2 = \frac{1}{2}$

iv) $\log_4 8 = \frac{3}{2}$

2) $\log_a (a^3) + \log_a \left(\frac{1}{a}\right) = \log_a a^2 = 2$

3. a) $\log_a x = 2 \log_a 6 - \log_a 3$
 $= \log_a 6^2 - \log_a 3$
 $= \log_a \frac{6^2}{3}$
 $= \log_a 12$

b) $\log_a y + \log_a 5 = 7$
 $a^7 = 5y$
 $\frac{a^7}{5} = y$

4. a) i) ~~$\log_a 1$~~ $\log_a 1 = 0$

ii) $\log_a a = 1$

b) $\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$
 $= \log_a \frac{5 \times 6}{1.5}$
 $= \log_a 20$

$$5. a) \log_a n = \log_a 3 + \log_a (2n+1)$$

$$\log_a n = \log_a 3 + \log_a (2n+1)$$

$$\log_a n = \log_a 3(2n+1)$$

$$n = 3(2n+1)$$

$$n = 6n + 3$$

$$-5n = 3$$

$$n = -\frac{3}{5}$$

$$b) i) \log_a x = 3$$

$$a^3 = x$$

$$ii) \log_a y - 3 \log_a 2 = 4$$

$$a^4 = \frac{y}{8}$$

$$y = 8a^4$$

$$xy = 8a^4 \times a^3$$

$$= 8a^7$$

$$b) a) \log_a x = \log_a 16 - \log_a 2$$

$$= \log_a \frac{16}{2}$$

$$= \log_a 8$$

$$b) \log_a y = 2 \log_a 3 + \log_a 4 + 1$$

$$= \log_a 3^2 + \log_a 4 + 1$$

$$= \log_a 36 + 1$$

$$\log_a y - \log_a 36 = 1$$

$$a^1 = \frac{y}{36}$$

$$y = 36a$$

$$(7) a) i) \sqrt{125} = 5^r$$

$$P = 3/2$$

$$ii) 2x = 3/2$$

$$x = 3/4$$

$$b) 3^{2x-1} = 0.05$$

$$\log_3 0.05 = 2x - 1$$

$$\frac{\log_3 0.05 + 1}{2} = x = -0.8634$$

$$c) \log_a x = 2(\log_a 3 + \log_a 2) - 1$$
$$= \log_a 3^2 + \log_a 2^2 - 1$$

$$\log_a x - \log_a 3^2 - \log_a 2^2 = -1$$

$$a^{-1} = \frac{x}{3^2 \times 2^2}$$

$$\frac{36}{a} = x$$

$$(8) a) i) \log_a 4 + \log_a 10 = \log_a 40$$

$$ii) \log_a 16 - \log_a 2 = \log_a 8$$

$$iii) 3 \log_a 5 = \log_a 125$$

$$b) \log_{1.5} 7.5 = 3x$$

$$x = \frac{\log_{1.5} 7.5}{3}$$

$$= 1.66$$

$$c) \log_2 p = m \text{ and } \log_8 q = n$$

$$p = 2^m, q = 8^n = 2^{3n}$$

$$pq = 2^m \times 2^{3n}$$

$$= 2^{3mn}$$

9. a) i) $\log_9 x = 0, x = 1$

ii) $\log_9 x = \frac{1}{2}, x = 3$

b) $2 \log_a n = \log_a 18 + \log_a (n-4)$

~~$\log_a n^2 = \log_a 18 + \log_a (n-4)$~~

~~$n^2 = 18 + n - 4$~~

~~$n^2 - n - 14 = 0$~~

$\log_a n^2 = \log_a 18(n-4)$

$n^2 - 18n + 72 = 0$

$(n-12)(n-6) = 0$

$n = 12 \text{ or } 6$

10. a) $\log_a N - \log_a x = \frac{3}{2}$

$a^{3/2} = \frac{aN}{x}$

~~$x = \frac{aN^{3/2}}{N}$~~

$x = \frac{N}{a^{3/2}}$

11. a) $\log_a b = c, a^c = b$

b) $2 \log_2 (x+7) - \log_2 (x+5) = 3$

$\log_2 \frac{(x+7)^2}{x+5} = 3$

$2^3 = \frac{x^2 + 14x + 49}{x+5}$

$8(x+5) = x^2 + 14x + 49$

$0 = x^2 + 6x + 9$

$0 = (x+3)^2$

$x = -3$

$$(12. a) \quad 2 \log_k x - \log_k 5 = 1$$

$$\log_k \frac{x^2}{5} = 1$$

$$k = \frac{x^2}{5}$$

$$\sqrt[3]{k} = \sqrt[3]{\frac{x^2}{5}}$$

$$b) \quad \log_a y = \frac{3}{2} \quad \log_4 a = b+2$$

$$y = a^{3/2} \quad a = 4^{b+2}$$

$$y = (4^{b+2})^{3/2}$$

$$= 2^{3b+2}$$

$$p = 3b+2$$